

ME 201

Thermodynamics

Exam #3 Solution

Problem 1

Steam at 0.5 MPa and 350°C is used to fill a 0.1 m³ tank, which is initially empty. After filling, the tank is cooled to 50°C and the contents become saturated liquid. Determine

- (a) the heat transfer required
- (b) the mass of water added to the tank

Solution:

System Type: Transient
Working Fluid: Compressible
Process: Constant total volume
State 1: Fixed
State 2: Fixed
State 3: Fixed

$$W_{\text{bnd}} = 0$$

$$W_{\text{sh}} = 0$$

$$Q = ???$$

conserv. of mass: $m_2 = m_3$

$$1^{\text{st}} \text{ Law: } m_2 u_2 = m_3 h_3 + Q$$

| State 1 (initial) | State 2 (final) | State 3 (inlet) |
|----------------------|----------------------------------------|-----------------------------------|
| T_1 | $T_2 = 50^\circ\text{C}$ | $T_3 = 350^\circ\text{C}$ |
| P_1 | $P_2 = 12.352 \text{ kPa}$ | $P_3 = 0.5 \text{ MPa}$ |
| u_1 | $u_2 = 209.33 \text{ kJ/kg}$ | $h_3 = 3168.1 \text{ kJ/kg}$ |
| $m_1 = 0 \text{ kg}$ | $m_2 = \mathbf{98.81 \text{ kg}}$ | $m_3 = \mathbf{98.81 \text{ kg}}$ |
| | $v_2 = 0.001012 \text{ m}^3/\text{kg}$ | |

Italicized values are from steam tables. **Bold** values are calculated.

Approach: We go to the steam tables and read the remaining properties for states 2 and 3. The mass in final state can then be calculated from V_3/v_3 . Finally the first law is used to calculate Q .

First we go to the air tables and find

$$u_2 = 209.33 \text{ kJ/kg}, v_2 = 0.001012 \text{ m}^3/\text{kg}, \text{ and } h_3 = 3168.1 \text{ kJ/kg}$$

Next we calculate the final mass from

$$m_2 = \frac{V_2}{v_2} = \frac{(0.1)}{(0.001012)} = 98.81 \text{ kg}$$

Conservation of mass then gives

$$m_3 = m_2 = 98.81 \text{ kg}$$

Using the 1st law to solve for the heat transfer we have

$$Q = m_2 u_2 - m_3 h_3 = (98.81)(209.33 - 3168.1) = -292,369 \text{ kJ}$$

Problem 2

In the Kentucky Derby a racehorse exerts a force of 36 Nt over a distance of 1900 m.

Model the racehorse as a heat engine operating under the following conditions

Net work is the force exerted times the distance traveled ($W_{\text{net}} = F \times d$)

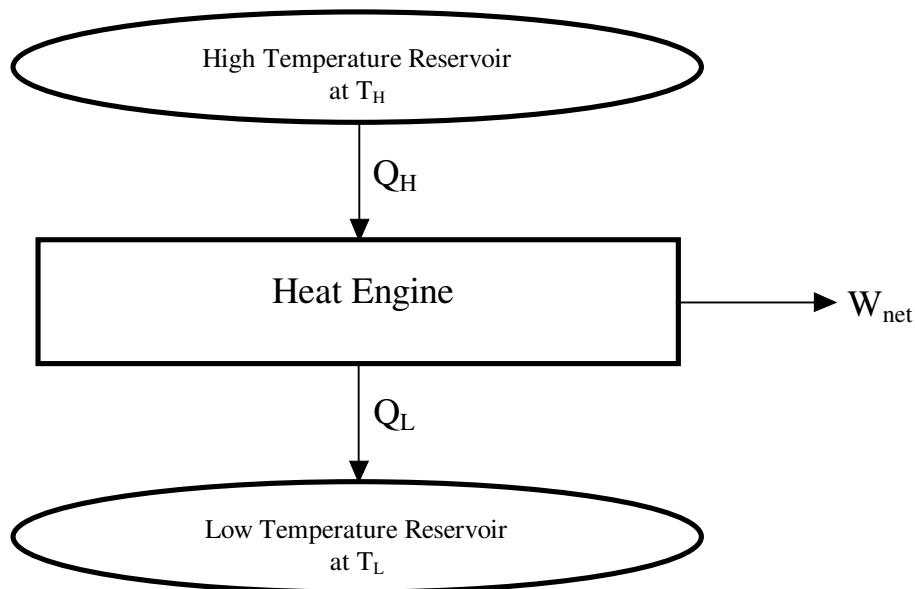
Low temperature heat reservoir is the environment at 20°C

High temperature heat reservoir is the internal temperature of the horse at 55°C

If the heat from the high temperature heat source comes from the digestion of grain with an energy content of 53 kJ/kg, determine the minimum amount of grain required for the horse to run the Derby.

Solution:

We begin with our interaction diagram for a heat engine.



Our appropriate equations are:

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{H}}}$$

$$Q_{\text{H}} = W_{\text{net}} + Q_{\text{L}}$$

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}}$$

We have been given the following conditions:

$$T_{\text{H}} = 55^{\circ}\text{C} = 328 \text{ K}$$

$$T_{\text{L}} = 20^{\circ}\text{C} = 293 \text{ K}$$

$$W_{\text{net}} = F \times d = (36)(1900) = 68,400 \text{ J} = 68.4 \text{ kJ}$$

The energy for Q_{H} must come from the grain so that

$$Q_{\text{H}} = m_{\text{grain}} \times 53 \text{ kJ/kg}$$

Then the problem reduces to determine Q_{H} . We begin by noting that since we want the minimum amount of grain, we can use our Carnot efficiency or

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{293}{328} = 0.1067$$

Then Q_{H} can be calculated

$$Q_{\text{H}} = \frac{W_{\text{net}}}{\eta_{\text{th}}} = \frac{68.4}{0.1067} = 641 \text{ kJ}$$

Finally

$$m_{\text{grain}} = \frac{Q_{\text{H}}}{(53 \text{ kJ/kg})} = \frac{(641)}{(53)} = 12.09 \text{ kg}$$

Problem 3

Air is expanded in a closed system from a state where the pressure is 500 kPa and the temperature is 285 K to a final state at 200 kPa and 450 K. Can this process occur adiabatically? If yes, determine the work per mass. If no, determine the direction of the heat transfer.

Solution:

We need to determine whether

$$(\Delta S)_{\text{universe}} \geq 0$$

holds for this situation. Since we have a closed system we can write this equivalently as

$$\begin{aligned} (\Delta S)_{\text{universe}} &= (\Delta S)_{\text{system}} + (\Delta S)_{\text{surrounds}} \geq 0 \\ &= m(s_2 - s_1) - \frac{Q_{\text{sys}}}{T_{\text{surr}}} \geq 0 \end{aligned}$$

Assuming that the process is adiabatic, we have

$$Q_{\text{sys}} = 0$$

Treating air as an ideal gas we have

$$s_2 - s_1 = \phi_2 - \phi_1 - R \ln(P_2/P_1)$$

Going to the air tables we find

$$\phi_2 = 2.11161 \text{ kJ}/(\text{kg}\cdot\text{K})$$

$$\phi_1 = 1.65055 \text{ kJ}/(\text{kg}\cdot\text{K})$$

Then substituting gives

$$s_2 - s_1 = 2.11161 - 1.65055 - (0.287)\ln(200/500) = 0.724035 \text{ kJ}/(\text{kg}\cdot\text{K})$$

Hence the process can be adiabatic. The work can be calculated from the 1st law or

$$w = u_1 - u_2$$

Going to the air tables we find

$$u_2 = 322.62 \text{ kJ}/\text{kg}$$

$$u_1 = 203.33 \text{ kJ}/\text{kg}$$

Then substituting gives

$$w = 203.33 - 322.62 = -119.29 \text{ kJ}/\text{kg}$$