ME 201
Thermodynamics

Exam 3 Solutions

Directions: Open notes (including course web postings). No books, computers, or phones. Any calculator is fair game.

Problem 1
An inventor claims to have developed a device that has an inlet flow of air at 290 K, 300 kPa, and 5 kg/s and two outlet flows, air at 330 K and 270 kPa and air at 270 K and 270 kPa. He claims that there is no work or heat transfer involved in the device. Evaluate this claim. The temperature of the surroundings may be taken to be 298 K.

Solutions:
To evaluate this claim we will want to calculate the rate of entropy production for the process and see if it is greater than or equal to zero, or
\[ S_{\text{prod}} = \dot{m}_3 s_3 + \dot{m}_2 s_2 - \dot{m}_1 s_1 \]

Our states our fixed, but to determine our mass flow rates we need to satisfy conservation of mass and conservation of energy. Hence, we will need to work a first law problem

System: Control Volume
Fluid: Air (ideal gas)
Process: \( \dot{Q} = 0 \) and \( \dot{W} = 0 \)

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 = 300 \text{ kPa} )</td>
<td>( P_2 = 270 \text{ kPa} )</td>
<td>( P_3 = 270 \text{ kPa} )</td>
</tr>
<tr>
<td>( T_1 = 290 \text{ K} )</td>
<td>( T_2 = 330 \text{ K} )</td>
<td>( T_3 = 270 \text{ K} )</td>
</tr>
<tr>
<td>( \dot{m}_1 = 5 \text{ kg/s} )</td>
<td>( \dot{m}_2 = 1.664 \text{ kg/s} )</td>
<td>( \dot{m}_3 = 3.336 \text{ kg/s} )</td>
</tr>
<tr>
<td>( \phi_1 = 6.6847 \text{ kJ/(kg} \cdot \text{K)} )</td>
<td>( \phi_2 = 6.8152 \text{ kJ/(kg} \cdot \text{K)} )</td>
<td>( \phi_3 = 6.6128 \text{ kJ/(kg} \cdot \text{K)} )</td>
</tr>
<tr>
<td>( h_1 = -8.07 \text{ kJ/kg} )</td>
<td>( h_2 = 32.31 \text{ kJ/kg} )</td>
<td>( h_3 = -28.22 \text{ kJ/kg} )</td>
</tr>
</tbody>
</table>

**Bold** values are calculated, *italicized* values are from air tables.

Our conservation of mass is given by
\[ \dot{m}_1 = \dot{m}_2 + \dot{m}_3 \]
and our conservation of energy is given by
\[ \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 0 \]

Substituting for \( \dot{m}_3 \) from mass into our energy equation we have
\[ \dot{m}_1 h_1 - \dot{m}_2 h_2 - (\dot{m}_1 - \dot{m}_2) h_3 = 0 \]

Now solving for \( \dot{m}_2 \)
\[
\dot{m}_2 = \frac{\dot{m}_1 (h_1 - h_3)}{h_2 - h_3} = \frac{(5)(-8.07 - (-28.22))}{(32.31 - (-28.22))} = 1.664 \text{ kg/s}
\]

and
\[
\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 5 - 1.664 = 3.336 \text{ kg/s}
\]

So we could now calculate our rate of entropy production, except we need to have entropy changes. To achieve this we substitute for \( \dot{m}_1 \) from conservation of mass to write
\[
\dot{S}_{\text{prod}} = \dot{m}_3 s_3 + \dot{m}_2 s_2 - (\dot{m}_2 + \dot{m}_3) s_1
\]

Now factoring the mass flow rates
\[
\dot{S}_{\text{prod}} = \dot{m}_3 (s_3 - s_1) + \dot{m}_2 (s_2 - s_1)
\]

For air as an ideal gas
\[
\dot{S}_{\text{prod}} = \dot{m}_3 \left( \frac{\phi_3 - \phi_1 - R \cdot \ln \frac{P_3}{P_1}}{P_1} \right) + \dot{m}_2 \left( \frac{\phi_2 - \phi_1 - R \cdot \ln \frac{P_2}{P_1}}{P_1} \right)
\]
\[
= (3.336) \left( 6.6128 - 6.6847 - (0.287)\ln \frac{270}{300} \right) + (1.664) \left( 6.8152 - 6.6847 - (0.287)\ln \frac{270}{300} \right)
\]
\[
= 0.1296 \text{ kW/K}
\]

So the device is possible.

**Problem 2**

Two kilograms of Refrigerant-12 is contained in a piston-cylinder system. It is initially at 150 kPa and 0°C and is compressed to saturated vapor at 0°C. The heat transfer from the cylinder is reported to be 30 kJ. Can this value be correct? What is the minimum work input required to carry out this process? The temperature of the surroundings may be taken to be 298 K.

**Solution:**

We need to determine whether
\[
(\Delta S)_{\text{universe}} \geq 0
\]
holds for this situation. Since we have a closed system we can write this equivalently as
\[
(\Delta S)_{\text{universe}} = (\Delta S)_{\text{system}} + (\Delta S)_{\text{surr}} \geq 0
\]
\[
= m(s_2 - s_1) \cdot \frac{Q_{\text{sys}}}{T_{\text{surr}}} \geq 0
\]

To evaluate this we will need to perform a 1\textsuperscript{st} law analysis. Setting up our table

<table>
<thead>
<tr>
<th>System: Closed System</th>
</tr>
</thead>
</table>

| Fluid: Refrigerant-12 (phase change substance) |

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(_1) = 0°C</td>
<td>T(_2) = 0°C</td>
</tr>
<tr>
<td>P(_1) = 150 kPa</td>
<td>P(_2) = 308.5 kPa</td>
</tr>
<tr>
<td>u(_1) = 172.80 kJ/kg</td>
<td>u(_2) = 170.43 kJ/kg</td>
</tr>
<tr>
<td>s(_1) = 0.7549 kJ/(kg \cdot K)</td>
<td>s(_2) = 0.6965 kJ/(kg \cdot K)</td>
</tr>
</tbody>
</table>

*Italicized* values are from R-12 tables

\[
m = 2 \text{ kg}
\]

State 1: Fixed

State 2: Fixed

W: UNKNOWN

Q = -30 kJ
1st Law: \( m(u_2 - u_1) = Q - W \)

Since both states are fixed we can go to the R-12 tables and find \( u \) and \( s \) for each state. Now calculating the entropy change of the universe

\[
(\Delta S)_{universe} = m(s_2 - s_1) - \frac{Q_{sys}}{T_{surr}}
\]

\[= (2)(0.6965 - 0.7549) - \frac{30}{298} = -0.01613 \text{kJ/K} \]

which does violate our 2nd law, so that the heat transfer measurement cannot be correct. To determine the minimum work, we must have a reversible process that satisfies

\[(\Delta S)_{universe} = 0\]

This allows us to calculate our \( Q \) from

\[m(s_2 - s_1) - \frac{Q_{sys}}{T_{surr}} = 0\]

or solving for \( Q_{sys} \)

\[Q_{sys} = T_{surr}m(s_2 - s_1)\]

\[= (298)(2)(0.6965 - 0.7549) = -34.81 \text{kJ} \]

Then the work may be calculated from the 1st law

\[W = -m(u_2 - u_1) + Q = -(2)(170.43 - 172.80) + (-34.81)\]

\[= -39.55 \text{kJ} \]

**Problem 3**

Determine the work per mass output of an adiabatic turbine with isentropic efficiency 0.83 that has a steam input of 15 MPa and 650°C and an outlet pressure of 325 kPa.

**Solution:**

The actual work will be calculated from the isentropic efficiency by

\[w_{act} = \eta_s w_{ideal}\]

Hence we need to calculate the ideal work. We begin by setting up our table and working the 1st law problem

System Type: Control Volume (Turbine)

Working Fluid: Steam (compressible Substance)

Process: Isentropic (ideal) Adiabatic (actual) with \( \eta_s = 0.83 \)

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2s</th>
<th>State 2a</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 ) = 650°C</td>
<td>( T_{2s} ) = 81.35 °C</td>
<td>( T_{2a} ) = 81.35 °C</td>
</tr>
<tr>
<td>( P_1 ) = 15 MPa</td>
<td>( P_{2s} ) = 350 kPa</td>
<td>( P_{2a} ) = 350 kPa</td>
</tr>
<tr>
<td>( h_1 ) = 370.647 kJ/kg</td>
<td>( h_{2s} ) = 2370.39 kJ/kg</td>
<td>( h_{2a} ) = 2597.52 kJ/kg</td>
</tr>
<tr>
<td>( s_1 ) = 6.8167 kJ/(kg·K)</td>
<td>( s_{2s} ) = 6.8167 kJ/(kg·K)</td>
<td>( s_{2a} ) = 7.4573 kJ/(kg·K)</td>
</tr>
<tr>
<td>phase: sup.vap.</td>
<td>phase: 2 phase with ( x_{2s} = 0.88 )</td>
<td>phase: 2 phase with ( x_{2a} = 0.979 )</td>
</tr>
</tbody>
</table>

*Italicized* values are from steam tables. *Bold* values are calculated.

State1: Fixed
State2s: UNKNOWN
State 2a: UNKNOWN  
\( \dot{Q} = 0 \)
\( \dot{W}_{sh} = \text{UNKNOWN} \)

1st Law: \( \dot{m}[h_1 - h_2] = \dot{W}_{sh} \)

Approach: To fix the state 2s we will use our process description which will give us our entropy at state 2. The remaining properties can then be determined from the steam tables. The ideal power is then calculated from the 1st law. The definition of isentropic efficiency will be used to calculate the actual power. The first law can then be used to calculate the actual exit enthalpy, which will fix state 2a and the properties can then be determined from the steam tables.

We start by completing our information for state 1. Going to the steam tables we find that we have superheated vapor with 
\( h_1 = 3706.47 \text{ kJ/kg} \) and \( s_1 = 6.8167 \text{ kJ/(kg} \cdot \text{K}) \)

Since we have an isentropic process the \( s \) at state 2s is given by 
\( s_{2s} = s_1 = 6.8167 \text{ kJ/(kg} \cdot \text{K}) \)

Then from the steam tables at 350 kPa we find that we have a two phase mixture with quality 
\[ x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{6.8167 - 1.7266}{6.9386 - 1.7266} = 0.977 \]

The enthalpy is then 
\( h_{2s} = h_f + x_{2s}h_{fg} = 583.98 + (0.977)(2147.66) = 2681.41 \text{ kJ/kg} \)

and the temperature is 
\( T_{2s} = 138.87^\circ\text{C} \)

Next we will calculate our ideal specific work from the 1st law 
\( w_{\text{ideal}} = h_1 - h_{2s} = 3706.47 - 2681.41 = 1025.06 \text{ kJ/kg} \)

Then our actual specific work is 
\[ w_{\text{act}} = \eta_w w_{\text{ideal}} = (0.83)(1025.06) = 850.8 \text{ kJ/kg} \]

Our actual exit enthalpy is 
\( h_{2a} = h_1 - w_{\text{act}} = 3706.47 - 850.8 = 2855.67 \text{ kJ/kg} \)

which gives us superheated vapor.