ME 201
Thermodynamics

Exam #2
Solution

Problem 1
Steam at 300 kPa with quality 0.96316 passes through a valve to convert it to saturated vapor. Determine the exit pressure required.
Solution: We begin with our template.

Substance Type: Compressible
System Type: Control Volume
Device: Valve
Process: Isenthalpic
Inlet State: Fixed
Outlet State: UNKNOWN
$q = 0$
$w_{sh} = 0$
1st Law: $h_2 - h_1 = 0$

State 1	State 2
T_1 = *133.52ºC*	T_2 = *81.31ºC*
P_1 = 300 kPa	P_2 = *50 kPa*
h_1 = *2645.2 kJ/kg*	h_2 = **2645.2 kJ/kg**
Phase: 2 phase with $x_1 = 0.96316$	Phase: sat.vap

Italicized values are from steam tables. **Bold** values are calculated.

Approach: Go to the steam tables and get h_f and h_g at 300 kPa. Use the quality to calculate h_1. Set $h_2 = h_1$. Go to the saturation pressure table and find the pressure that gives an $h_g = h_2$.

At 300 kPa, we have

$h_f = 561.43$ kJ/kg and $h_g = 2724.9$ kJ/kg

Then we can calculate

$$h_1 = (1-x_1)h_f + x_1 h_g = (1-0.96316)(561.43) + (0.96316)(2724.9)$$
$$= 2645.2 \text{ kJ/kg}$$

Since our valve is isenthalpic

$h_2 = h_1 = 2645.2$ kJ/kg

Now going to the saturation pressure table we find

$P_2 = 50$ kPa

Problem 2

A piston-cylinder device contains 0.001 kg of CO_2 at 750 kPa and 1340 K. The device undergoes a polytropic process with polytropic exponent -0.3 or

$$PV^{-0.3} = \text{constant}$$

which decreases the pressure to 550.2 kPa. Determine the boundary work and heat transfer in kJ for this process.

Solution: We begin with our template.

Substance Type: Ideal Gas
System Type: Closed System
Process: Polytropic with n=-0.3
Initial State: Fixed
Final State: UNKNOWN
Q:UNKNOWN

$$W_{sh} = 0$$

$$W_{bnd} = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$

Conserv. of mass: $m_1 = m_2 = 0.001$ kg

1st Law: $m(u_2 - u_1) = Q - W_{bnd}$

<u>State 1</u>	<u>State 2</u>
$T_1 = 1340$ K	$T_2 =$ *350 K*
$P_1 = 750$ kPa	$P_2 = 550.2$ kPa
$u_1 =$ *1151.4 kJ/kg*	$u_2 =$ *191.75 kJ/kg*
$v_1 =$ *0.3375 m³/kg*	$v_2 =$ *0.1202 m³/kg*
$V_1 =$ *3.375 x 10⁻⁴ m³*	$V_2 =$ **1.202x 10⁻⁴ m³**
$m_1 = 0.001$ kg	$m_2 = 0.001$ kg

Italicized values are from air tables or ideal gas relation. **Bold** values are calculated.
Approach: At state 1, go to the air tables and get u_1. Use the ideal gas relation to calculate v_1. Calculate V_1 from v_1 and m_1. Use polytropic relation to calculate V_2. Calculate v_2 from V_2 and m_2. Calculate T_2 from ideal gas relation. Read u_2 from air tables. Calculate the boundary work from PdV. Calculate the heat transfer from the 1^{st} law.

From the air tables for state 1

$$u_1 = 50{,}672 \text{ kJ/kgmol} = (50{,}672)/(44.01) = 1151.4 \text{ kJ/kg}$$

Using the ideal gas law

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.1889)(1340)}{(750)} = 0.3375\,\text{m}^3/\text{kg}$$

Then

$$V_1 = m_1 v_1 = (0.001)(0.3375) = 3.375 \text{ x } 10^{-4} \text{ m}^3$$

At state 2 we know the pressure and have a polytropic process so that

$$P_2 V_2^n = P_1 V_1^n \text{ or solving}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{1/n} \text{ or substituting}$$

$$V_2 = (3.375 \text{ x } 10^{-4}) \left(\frac{750}{550.2} \right)^{-1/0.3} = 1.202 \text{ x } 10^{-4} \text{ m}^3$$

Then

$$v_2 = \frac{V_2}{m_2} = \frac{1.202 \text{ x } 10^{-4}}{0.001} = 0.1202 \text{ m}^3$$

We can then determine the temperature at state 2 from

$$T_2 = \frac{P_2 v_2}{R} = \frac{(550.2)(0.1202)}{(0.1889)} = 350 \text{ K}$$

From the air tables for state 2

$$u_2 = 8{,}439 \text{ kJ/kgmol} = (8{,}439)/(44.01) = 191.75 \text{ kJ/kg}$$

We can now use our equation for the boundary work

$$W_{bnd} = \frac{P_2 V_2 - P_1 V_1}{1 - n} = \frac{(550.2)(1.202 \text{ x } 10^{-4}) - (750)(3.375 \text{ x } 10^{-4})}{1 - (-0.30)} = -0.1439 \text{ kJ}$$

From the 1st law we calculate the heat transfer

$$Q = m(u_2 - u_1) + W_{bnd} = (0.001)(191.75-1151.4) + (-0.1439) = -1.1035 \text{ kJ}$$

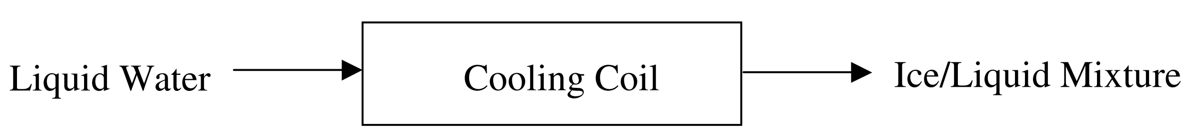
Problem 3

A proposed new ice maker is composed of two components. The first component is a cooling coil, while the second component is a freezing chamber. Water at 15ºC and 100 kPa enters the cooling coil at 0.05 kg/s and exits at 0ºC as a 50% - 50% mixture by mass of liquid and solid. Determine the heat transfer rate in kW for the cooling coil.

Hint: The enthalpy for subcooled liquid or solid can be calculated from the tables with

sub.liq.: $h_1 = h_f(\text{at } T_1) + v_f(\text{at } T_1) \text{ x } [P_1 - P_{sat}(\text{at } T_1)]$

sub.sol.: $h_1 = h_i(\text{at } T_1) + v_i(\text{at } T_1) \text{ x } [P_1 - P_{sat}(\text{at } T_1)]$



Solution: We begin with the cooling coil. Setting up our template,

Substance Type: Compressible
System Type: Control Volume
Device: Heat Device
Process: Constant Pressure
Inlet State: Fixed
Outlet State: UNKNOWN
q = ???
$w_{sh} = 0$
1st Law: $h_2 - h_1 = q$

State 1	State 2
T_1 = 15ºC	T_2 = *0ºC*
P_1 = 100 kPa	P_2 = *100 kPa*
h_1 = *63.08 kJ/kg*	h_2 = **2645.2 kJ/kg**
Phase: sub. liq.	Phase: 2 phase with $x_2 = 0.50$

Italicized values are from steam tables. **Bold** values are calculated.

Approach: With state 1 fixed, we can go to the steam tables and get h_f, v_f, and P_{sat} at 15ºC and then use the equation in the hint to calculate h_1. Next we use the constant pressure condition to fix state 2. We calculate the liquid and solid enthalpies using the equation from our hint and then use the quality to calculate h_2. Use the 1st law to calculate the heat transfer rate.

At 15ºC we find

$h_f = 62.982$ kJ/kg, $v_f = 0.001001$ m^3/kg and $P_{sat} = 1.7057$ kPa

Then using our hint

$h_1 = h_f(\text{at } T_1) + v_f(\text{at } T_1) \times [P_1 - P_{sat}(\text{at } T_1)] = 62.982 + (0.001001)(100-1.7057)$
$= 63.08$ kJ/kg

At 0ºC we have

$h_f = 0.001$ kJ/kg, $v_f = 0.001000$ m^3/kg and $P_{sat} = 0.6117$ kPa

so that

$h_{2,\ell} = h_f(\text{at } T_2) + v_f(\text{at } T_2) \times [P_2 - P_{sat}(\text{at } T_2)] = 0.001 + (0.001000)(100-0.6117)$
$= 0.1004$ kJ/kg

and

$h_i = -333.40$ kJ/kg, $v_i = 0.001091$ m^3/kg and $P_{sat} = 0.61169$ kPa

so that

$h_{2,s} = h_i(\text{at } T_2) + v_i(\text{at } T_2) \times [P_2 - P_{sat}(\text{at } T_2)] = -333.40 + (0.001091)(100-0.61169)$
$= -333.292$ kJ/kg

Then

$h_2 = (0.5)h_{2,\ell} + (0.5)h_{2,s} = (0.5)(0.1004) + (0.5)(-333.292) = -166.596$ kJ/kg

Our 1st law then gives

$$\dot{Q} = \dot{m}(h_2 - h_1) = (0.05)(-166.596 - 63.08) = -11.48\ \text{kW}$$