The dependence of carrier lifetime on injection level has been measured in silicon power devices. As examples, the results of an Au-doped and an as-processed, not intentionally doped, specimen are given. The experimental results confirm the Shockley-Read-Hall recombination theory. The ratio of the capture cross-sections of the holes and electrons is calculated.

Even today, the recombination theory developed by Shockley and Read and Hall is the basis of all calculations and practical processes that attempt to control the carrier lifetime in silicon devices. In particular, in high-frequency and high-voltage devices, it is necessary to fix the rate of recombination in the structure. This means, in practice, that the lifetime, and its dependence on the temperature and injection level, must be known exactly. If one considers, for instance, the injection-level dependence of the lifetime in a forward-biased thyristor, the forward voltage drop is essentially determined by the high-injection-level lifetime \( \tau_n \). During the turnoff process, the lifetime at the beginning is still the high-level value, and this value then changes with the decreasing carrier concentration. Towards the end of the turnoff process, the low-level lifetime \( \tau_0 \) is reached. Also, for the dimensioning of the neutral region in the base of a thyristor under reverse bias, the low-level lifetime must be used.

Although, from the modelling of power devices, the technological importance of these questions is evident, since theoretical calculations of these interdependences have been available for a long time, very little experimental work in this field has been reported. With a few exceptions, only a small range of injection levels has been considered in all reported investigations. This is surprising, since, from the values of \( \tau_n \) and \( \tau_0 \), one can determine the capture cross-sections of the corresponding recombination centres, which, in turn, would give an experimental basis for a fundamental theoretical treatment of the recombination mechanisms in silicon.

Possibly the reason for this lack of experimental confirmation of the SRH theory is that it was only possible in a few cases to cover a sufficiently wide range of injection levels. In a previous paper, a method for measuring lifetimes using a pulsed electron beam was described that is suitable for studying directly the injection-level dependence of the lifetime.

Equal thermal velocities for the holes and electrons have been assumed.

Specimen 1 was prepared from a normally processed power device. It was not intentionally doped with any particular recombination centre. Therefore nothing further can be said about the nature of this centre.

The second specimen was prepared from a thyristor, which had been intentionally doped with Au to obtain fast-switching characteristics. Therefore the value \( \sigma_p/\sigma_n = 3.0 \) is characteristic of the Au recombination centre at \( T = 300 \text{ K} \). We believe that the accuracy of this result is considerably better than those previously presented in the literature.

There is a considerable difference in the ratio of the capture cross-sections between the Au in specimen 2 and the unknown centre in specimen 1. The resulting differences in injection-level dependence are also considerable. It is thus evident that the properties of the effective centre must in all cases be well known to produce the optimum device geometry.

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>( \rho )</th>
<th>( \tau_n )</th>
<th>( \tau_0 )</th>
<th>( \sigma_p/\sigma_n \approx \tau_0/\tau_n )</th>
<th>Nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen 1</td>
<td>220</td>
<td>23.5 ( \mu \text{s} )</td>
<td>1.5 ( \mu \text{s} )</td>
<td>15.7</td>
<td>Unknown</td>
</tr>
<tr>
<td>Specimen 2</td>
<td>40</td>
<td>750 ns</td>
<td>250 ns</td>
<td>3.0</td>
<td>Au</td>
</tr>
</tbody>
</table>

From the Shockley-Read-Hall recombination theory, the relation between the lifetime and the carrier concentration in n-type silicon at room temperature is given by

\[
\tau = \tau_0 + \tau_n \frac{\Delta n}{\Delta n + n_0}
\]

where \( \Delta n = \Delta \rho = \text{excess-carrier concentration} \)

\( n_0 = \text{electron concentration at thermal equilibrium} \)

\( \tau_n, \tau_0 = \text{lifetimes of holes and electrons, respectively,} \)

under the assumption that all centres are occupied by electrons or holes, respectively.
A technique is described for the tuning of a feedback-control system for a plant described by a dyadic transfer-function matrix. It is shown that the closed-loop structure can be modified using a frequency-independent feedback element, and that the stability of the modified system is unchanged.

A recent paper describes a technique for the feedback-control analysis of a class of linear multivariable systems whose behavior can be approximated by the transfer-function relationships

\[ y(s) = H(s)u(s) \]  

Here \( y(s) \) is the \( N \times 1 \) vector of output transforms, \( u(s) \) is the vector of control input transforms and \( H(s) \) is an \( N \times N \) transfer-function matrix of the form

\[ H(s) = \sum_{j=1}^{N} h_j(s) \alpha_j \beta_j^* \]  

and \( H^{-1}(0) \) exists, where \( \{\alpha_j\} \) and \( \{\beta_j^*\} \) are sets of real frequency-independent, linearly independent column and row vectors, respectively. The coefficients \( h_j(s) \) are scalar transfer functions. The feedback structure described has a sound practical motivation, as certain nuclear-reactor spatial control configurations have the above structure.\(^1,2\)

Using a conventional negative-feedback-controller configuration,\(^1\) it is shown that the closed-loop system can be designed to be stable and fail-safe using a unit feedback matrix \( K(s) \) and a forward path-controller matrix of the form

\[ G(s) = H^{-1}(0) \sum_{j=1}^{N} g_j(s) \alpha_j \gamma_j^* \]  

where the coefficients \( g_j(s) \) are scalar transfer functions, and \( \{\gamma_j^*\} \) are a set of linearly independent row vectors satisfying

\[ \gamma_j^* \alpha_k = \delta_{jk} \]  

With these controller factors, the closed-loop-system transfer-function matrix takes the form

\[ (I + H(s)G(s))^{-1} H(s)G(s) = \sum_{j=1}^{N} \mu_j(s) \alpha_j \gamma_j^* \]  

where \( I \leq j \leq N, \) and

\[ \mu_j(s) = \frac{h_j(s)g_j(s)}{1 + h_j(s)g_j(s)} \]  

so that the closed-loop-system stability is governed by the stability of the single-input single-output subsystems \( \mu_j(s) \). The transient response of the system is governed by the subsystem responses and the form of the vectors \( \{\alpha_j\} \).

Consider a feedback-control system designed using the above concepts. The closed-loop response (eqn. 5) is given as a linear combination of the natural motions \( \{\mu_j(s) \alpha_j\} \), and the precise contribution of these natural motions to the output response is governed by the row vectors \( \{\gamma_j^*\} \). The transfer-function matrices \( g_j(s) \) are assumed to have been chosen from an available set to reach some compromise between the required stability margins and the closed-loop-system transient performance.

At the end of the design procedure, it may happen that an analysis of the natural motions \( \{\mu_j(s) \alpha_j\} \) indicates that an improved transient response could be obtained by modifying their contributions to the output response. In mathematical terms, an improved response may be obtained if the closed-loop-system transfer-function matrix takes the form

\[ \sum_{j=1}^{N} \mu_j(s) \alpha_j \gamma_j^* \]  

where \( \{\gamma_j^*\} \) is a set of \( N \) linearly independent frequency-independent row vectors. This system (eqn. 7) has the same stability properties as the system of eqn. 5, but, by assumption, it has an improved transient performance.

The problem investigated in this letter is how one can achieve the closed-loop response of eqn. 7 by a modification of the forward-path controller matrix \( G(s) \) (see eqn. 3) and the use of a nonunit feedback matrix \( K(s) \).

As the \( \{\gamma_j^*\} \) are linearly independent, it is possible to construct a set of \( N \) linearly independent frequency-independent column vectors \( \{w_j\} \) such that

\[ v_j^* w_k = \delta_{jk} \]  

Consider the forward-path controller

\[ G_1(s) = H^{-1}(0) \sum_{j=1}^{N} g_j(s) \alpha_j v_j^* \]  

and the feedback matrix

\[ K_1(s) = \sum_{j=1}^{N} w_j \gamma_j^* \]  

then, as\(^4\)

\[ \beta_j^* H^{-1}(0) \alpha_k = \delta_{jk} \]  

and eqn. 8 holds, it follows that

\[ (I + H(s)G_1(s)K_1(s))^{-1} = \sum_{j=1}^{N} (I + h_j(s)g_j(s))^{-1} \alpha_j \gamma_j^* \]  

Also, using eqn. 11,

\[ H(s)G_1(s) = \sum_{j=1}^{N} h_j(s)g_j(s) \alpha_j v_j^* \]  

Multiplying eqn. 13 by eqn. 14, and using eqns. 4 and 6, the closed-loop transfer-function matrix of the modified system is

\[ (I + H(s)G_1(s)K_1(s))^{-1} H(s)G_1(s) = \sum_{j=1}^{N} \mu_j(s) \alpha_j v_j^* \]