ECE202 (Fall 2007)
HW # 7
Due: Friday, 10/19/2007

[100 pts]

(P. 1) [60 pts] For the circuit in Figure 1:

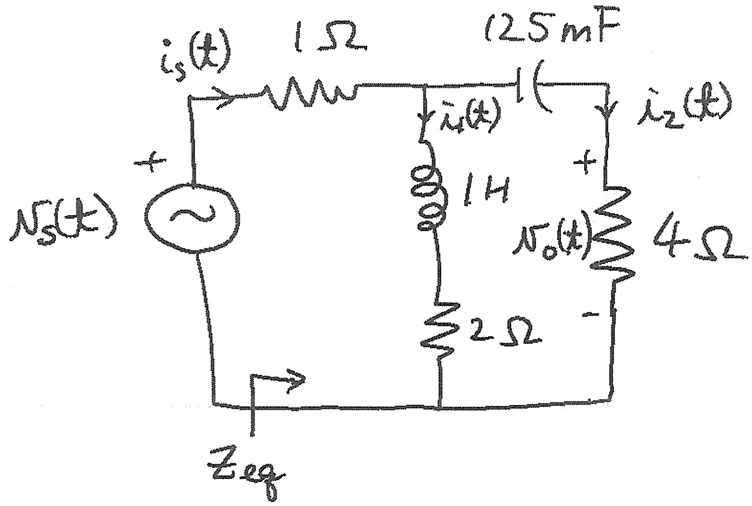


Figure 1 Circuit diagram for (P. 1)

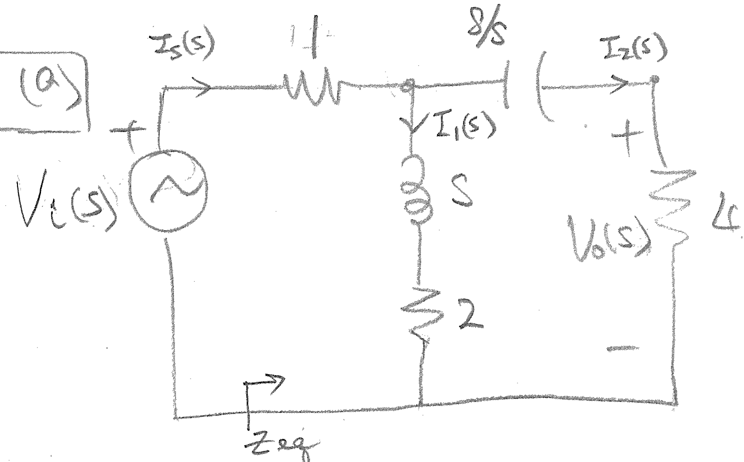
- (a) [11 pts] Transform and redraw the circuit in the Laplace domain (label all variables clearly).
- (b) [10 pts] Determine the equivalent impedance, Z_{eq}.
- (c) [18 pts] Identify the poles and zeros of Z_{eq} before and after simplification/cancellation. Plot the poles and zeros in the complex plane (label all your axes).
- (d) [6 pts] What are the corner frequencies for Z_{eq}?
- (e) [15 pts] Construct the Bode plots for Z_{eq}.

(P.2) [25 pts] For the circuit in Figure 1:

- (a) [5 pts] Determine the transfer function $H(s) = V_o(s) / V_s(s)$.
- (b) [6 pts] Identify the poles and zeros of the circuit from $H(s)$ and plot them in the complex plane (label all your axes).
- (c) [2 pts] What is/are the critical frequency/frequencies?
- (d) [10 pts] Construct the Bode plots for the circuit.
- (e) [2 pts] What type of gain response is this?

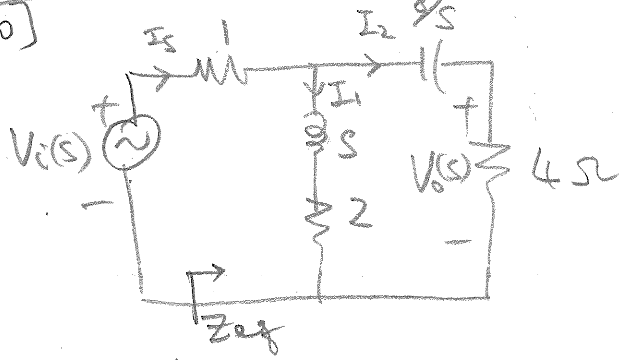
(P.3) [15 pts] Show that a series LR circuit is a lowpass filter if the output is taken across the resistor. Derive an expression for the corner (cutoff) frequency, ω_c. Calculate the corner frequency, f_c, if L = 2 mH and R = 10 kΩ.

P.1 (a) [12]



(1 pt each element)

P.1(b) [10] The equivalent impedance, Z_{eq}:



$$V_o(s) = I_2(s) \cdot 4 \quad \text{—— (1)}$$

current divider

$$I_2(s) = I_s(s) \cdot \frac{(s+2)}{\left(s+2+\frac{8}{s}+4\right)}$$

$$= I_s(s) \cdot \frac{(s+2)}{\left(s+\frac{8}{s}+6\right)}$$

$$= I_s(s) \frac{(s)(s+2)}{(s^2+6s+8)}$$

$$= I_s(s) \frac{(s)(s+2)}{(s+2)(s+4)}$$

$$I_2(s) = I_s(s)\left(\frac{s}{s+4}\right) \quad \text{—— (2)}$$

$$I_s(s) = \frac{V_i(s)}{Z_{eq}}$$

$$Z_{eq} = 1 + (s+2)//\left(\frac{8}{s}+4\right)$$

$$= 1 + \frac{(s+2)\left(\frac{8}{s}+4\right)}{s+2+\frac{8}{s}+4}$$

$$= 1 + \frac{8+\frac{16}{s}+4s+8}{s+\frac{8}{s}+6}$$

$$= 1 + \frac{4s^2+16s+16}{s^2+6s+8}$$

$$= \frac{s^2+6s+8+4s^2+16s+16}{s^2+6s+8}$$

pole-zero cancellation

$$= \frac{5s^2+22s+24}{s^2+6s+8} = \boxed{\frac{(5s+12)\cancel{(s+2)}}{(s+4)\cancel{(s+2)}}} = \boxed{\frac{5s+12}{s+4} = Z_{eq}}$$

Before 5 After 5

(2 pts for each element) (1 pt for labels)

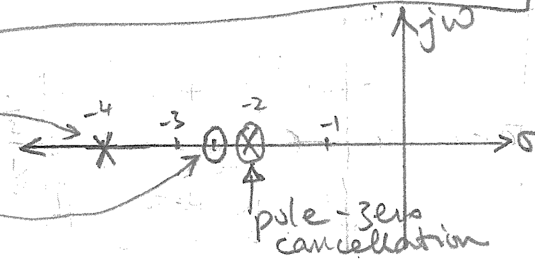
2/5

P.1(c)

$$Z_{eq}(s) = \frac{5s+12}{s+4}$$

$\boxed{\begin{array}{l} \text{zero @ } s = -2.4 \quad (s=-2) \\ \text{pole @ } s = -4 \quad (s=-2) \end{array}}$

this pole & zero cancel each other



$$Z_{eq}(j\omega) = \frac{5(j\omega)+12}{j\omega+4}$$

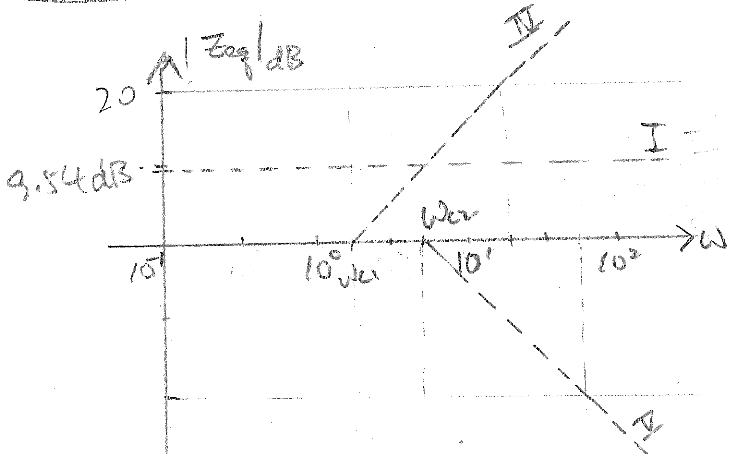
$$= \frac{12\left[1+j\omega\left(\frac{5}{12}\right)\right]}{4\left[1+j\omega\left(\frac{1}{4}\right)\right]}$$

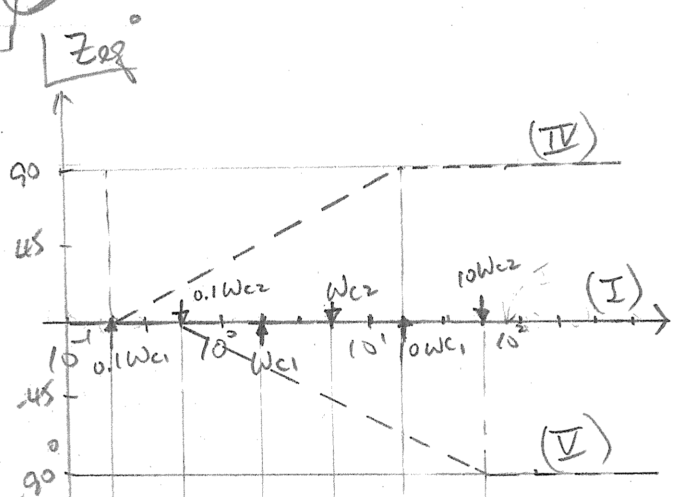
$$Z_{eq}(j\omega) = \underbrace{\left[3\right]}_{(I)}\underbrace{\left[1+j\omega\left(\frac{5}{12}\right)\right]}_{(IV)}\underbrace{\left[\frac{1}{1+j\frac{\omega}{4}}\right]}_{(V)}$$

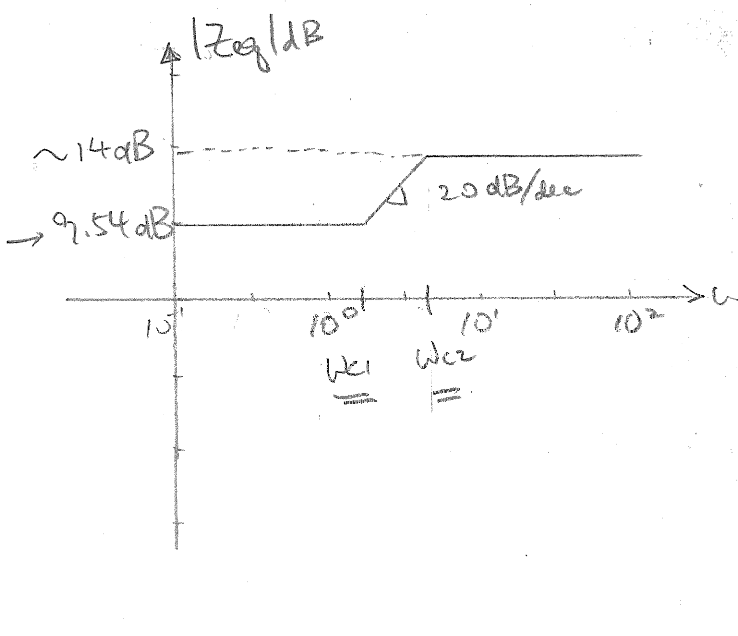
P.1(d)

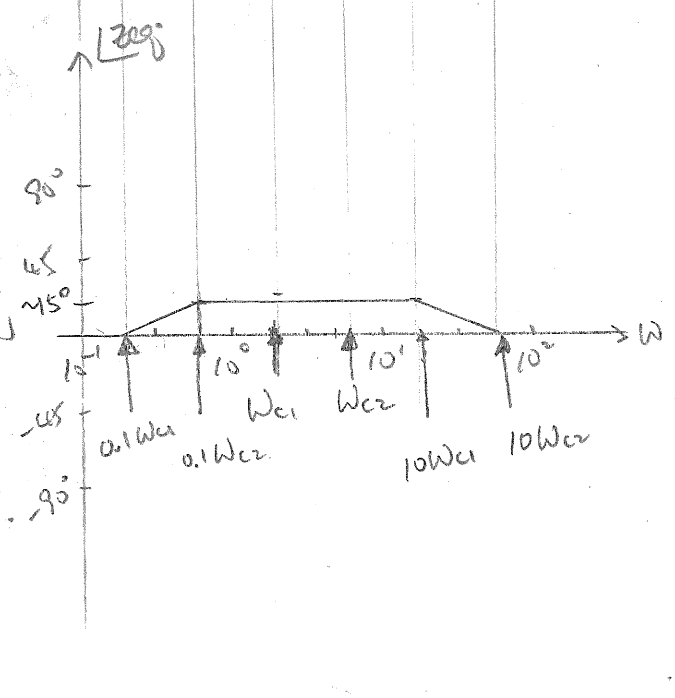
$\boxed{\begin{array}{l} \text{From (IV)}, \ \omega_{c1} = 2.4 \text{ rad s}^{-1} \\ \text{From (V)}, \ \omega_{c2} = 4 \text{ rad s}^{-1} \end{array}}$ (3) (3)

P.1(e)









P. 2(a) 5

From P.1 : $Z_{eq} = \dfrac{5s+12}{s+4}$

$$V_o(s) = I_2(s)\cdot 4 \quad \text{———} \quad (1)$$

$$I_2(s) = I_s(s)\left(\frac{s}{s+4}\right) \quad \text{———} \quad (2)$$

$$\therefore\ I_s(s) = \frac{V_s(s)}{Z_{eq}} = V_s(s)\,\frac{(s+4)}{(5s+12)} \quad \text{—} \quad (3)$$

sub (3) into (2) yields

$$I_2(s) = V_s(s)\,\frac{(s+4)}{(5s+12)}\cdot\frac{s}{(s+4)} \quad \text{—} \quad (4)$$

sub (4) into (1) yields

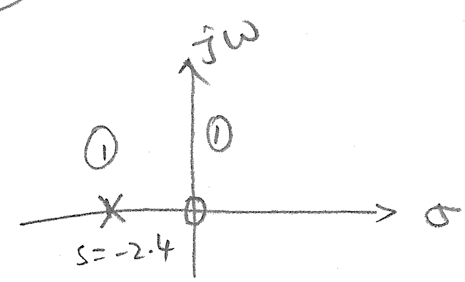
$$V_o(s) = I_2(s)\cdot 4 = V_s(s)\,\frac{s}{(5s+12)}\cdot 4$$

$$\therefore\ \boxed{H(s) = \frac{V_o(s)}{V_s(s)} = \frac{4s}{5s+12}} \quad 5$$

P. 2(b) 6

2 zero @ $s=0$

2 pole @ $s=-2.4$



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P.2(c)

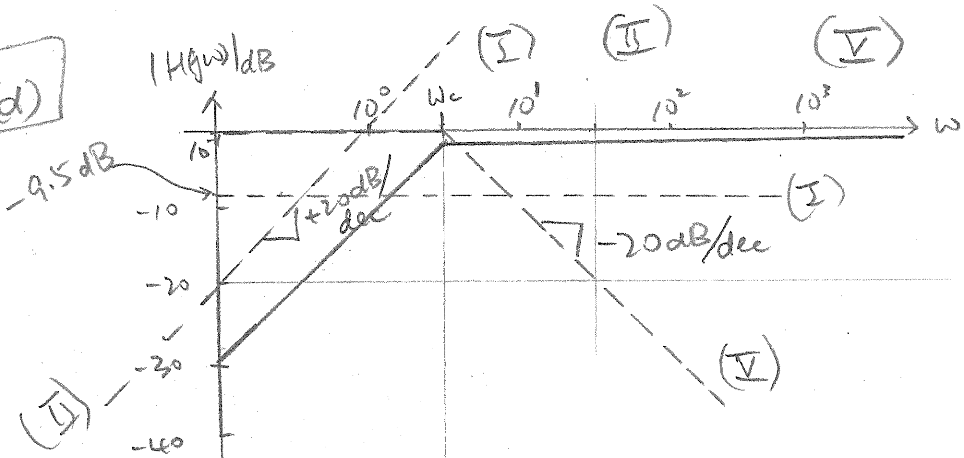
3pt

$$H(s)=\frac{4s}{12\left(1+s\frac{5}{12}\right)} \qquad H(j\omega)=\frac{4j\omega}{12\left(1+j\frac{\omega}{2.4}\right)}$$

$$H(j\omega)=\left[\tfrac{1}{3}\right]\left[j\omega\right]\left[\frac{1}{1+j\frac{\omega}{2.4}}\right] \rightarrow \boxed{\omega_c = 2.4 \text{ rads}^{-1}}$$

P.2(d)

10



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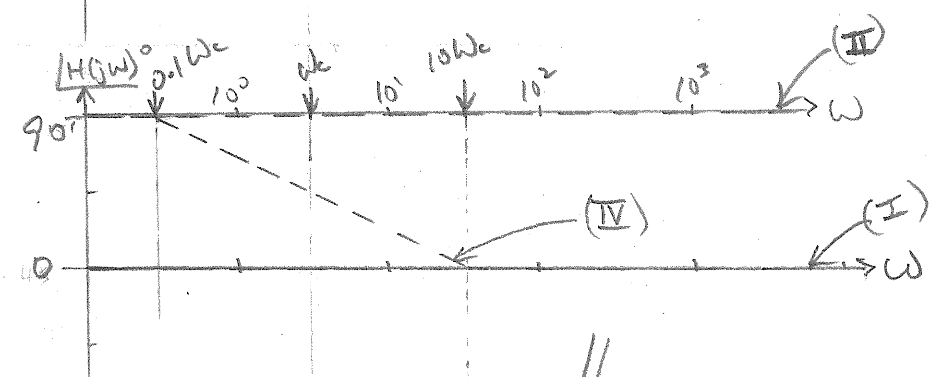
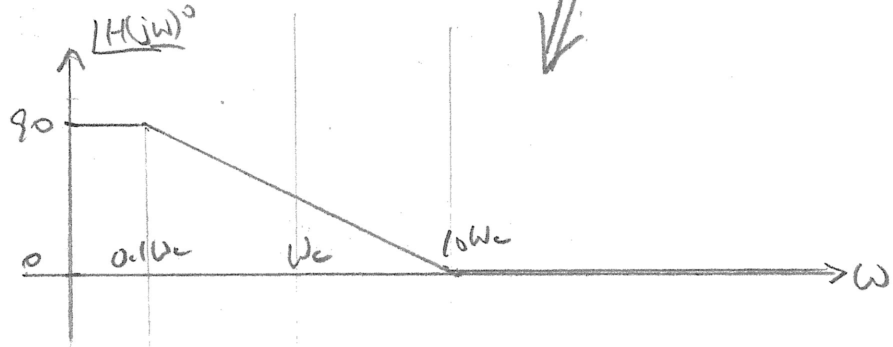




5

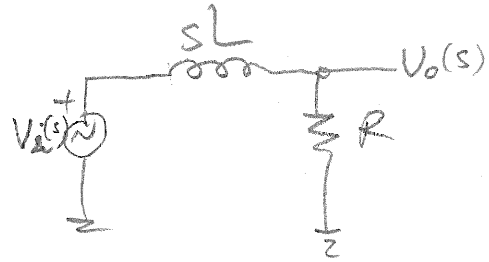
P.2(e)

2pt

High-pass filter w/ cutoff freq @ $\omega_c = 2.4 \text{ rads}^{-1}$

5/5

(P.3)



$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R+sL}$$

$$H(j\omega) = \frac{R}{R+j\omega L}$$ (5)

(a) at $\omega = 0$, $H(0) = 1$; $\omega = \infty$, $H(\infty) = 0$ } $H(s)$ is a lowpass filter!

(b) At the corner (cutoff) frequency.

$$|H(\omega_c)| = \frac{1}{\sqrt{2}}|H(0)|$$

i.e.,
$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\left(\frac{\omega_c L}{R}\right)^2}}$$

$$1 = \frac{\omega_c L}{R} \quad \text{or} \quad \boxed{\omega_c = \frac{R}{L}}$$ (5)

(c) $\omega_c = \frac{R}{L} = 2\pi f_c$

$$f_c = \frac{1}{2\pi}\frac{R}{L} = \frac{1}{2\pi}\frac{10\times10^3}{2\times10^{-3}}$$

$$\boxed{f_c = 796 \text{ kHz}}$$ (5)