ECE202 (Fall 2007)
HW # 6
Due: Friday, 10/12/2007

P.1 [10 pts] Solve Problem 9-11 in text.

P.2 [10 pts] Solve Problem 9-12 in text.

P.3 [20 pts] Solve Problem 9-13 in text.

P.4 [15 pts] Solve Problem 9-16 in text

P.5 [35 pts]

(a) [24 pts] Solve Problem 9-26 in text

(b) [10 pts] Verify your answers to part (a) by solving for $f_1(t)$ and $f_2(t)$ directly using MATLAB. (You must show your ML code to receive credit.)

# ECE202 Fall 2007 H.W. #6



(P. 1) Solve P9-11 in text

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$$f(t) = (5 - 2e^{-5t})u(t) - 3\cos[10(t-0.5)]\,u(t-0.5)$$

$$F(s) = \mathcal{L}\{f(t)\}$$

$$= \mathcal{L}\{(5-2e^{-5t})u(t)\}$$

$$-3\mathcal{L}\{\cos[10(t-0.5)]u(t-0.5)\}$$

$$= \frac{5}{s} - \frac{2}{s+5} - 3e^{-0.5s}\cdot\mathcal{L}\{\cos(10t)u(t)\}$$

$$\boxed{F(s) = \frac{3s+25}{s(s+5)} - 3e^{-(0.5)s}\left(\frac{s}{s^2+100}\right)}$$

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(P.2) Solve P9-12 in text

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(a) $f_1(t) = \frac{d}{dt}(10e^{-5t}\cos 20t)$

$$F_1(s) = \mathcal{L}\{f_1(t)\} = \mathcal{L}\left\{\frac{d}{dt}(10e^{-5t}\cos(20t))\right\}$$

$$= s \cdot \mathcal{L}\{10e^{-5t}\cos(20t)\} - 10$$

$$= \frac{10 \cdot s \cdot (s+5)}{(s+5)^2 + 20^2} - 10$$

$$= \frac{10s^2 + 50s}{s^2 + 10s + 425} - 10$$

$$= \frac{10s^2 + 50s - 10s^2 - 100s - 4250}{s^2 + 10s + 425}$$

$$= \frac{-50s - 4250}{s^2 + 10s + 425}$$

$$\boxed{F_1(s) = \frac{-50(s+85)}{s^2 + 10s + 425}}$$

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(P.2) (b) $f_2(t) = \int_0^t e^{-10x}\,dx + 10te^{-10t}\,u(t)$

$$F_2(s) = \mathcal{L}\{f_2(t)\}$$

$$= \mathcal{L}\left\{\int_0^t e^{-10x}\,dx + 10te^{-10t}\right\}$$

$$= \frac{1}{s}\mathcal{L}\{e^{-10t}\} + \mathcal{L}\{10te^{-10t}\}$$

$$= \frac{1}{s(s+10)} + \frac{10}{(s+10)^2}$$

$$= \frac{s+10+10s}{s(s+10)^2}$$

$$\boxed{F_2(s) = \frac{11s+10}{s(s+10)^2}}$$

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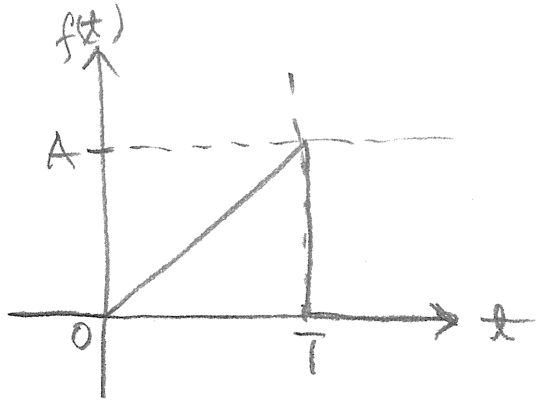
P. 3
(P9-13 in text)




*Note: If the simplified form is used as in Supple. Problem S9.1(a), you may receive credit for part (a) only (max = 8 pts). You won't be able to apply the translation property for part (b)

9-13(a) $f(t) = f_1(t) + f_2(t) + f_3(t)$

$f_1(t) = \frac{A}{T} t\, u(t)$

$f_2(t) = -\frac{A}{T}(t-T)\, u(t-T) = f_1(t-T)$

$f_3(t) = -A\, u(t-T)$

$\therefore \boxed{f(t) = \frac{A}{T} t\, u(t) - \frac{A}{T}(t-T)\, u(t-T) - A\, u(t-T)}$ * 8

9-13(b) $F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{f_1(t)\} + \mathcal{L}\{f_2(t)\} + \mathcal{L}\{f_3(t)\}$

$\mathcal{L}\{f_1(t)\} = \frac{A}{T}\frac{1}{s^2}$ (from Table 9-2, p.410 of text)

$\mathcal{L}\{f_2(t)\} = e^{-sT} F_1(s)$ (from Table 9-1 of text)

$= e^{-sT}\left(-\frac{A}{T}\right)\left(\frac{1}{s^2}\right)$

$\mathcal{L}\{f_3(t)\} = -Ae^{-sT}\mathcal{L}\{u(t)\} = -Ae^{-sT}\left(\frac{1}{s}\right)$

$\therefore F(s) = \frac{A}{T}\frac{1}{s^2} - e^{-sT}\frac{A}{Ts^2} - e^{-sT}\frac{A}{s}$ 2 + 2 + 2

$\boxed{F(s) = \frac{A}{Ts^2}\left(1 - e^{-sT} - sTe^{-sT}\right)}$

P. 3 (c) $F(s) = \int_0^\infty f(t) e^{-st} dt$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{f_1(t)\} + \mathcal{L}\{f_2(t)\} + \mathcal{L}\{f_3(t)\}$$

$$\mathcal{L}\{f_1(t)\} = \mathcal{L}\{\tfrac{A}{T} t\, u(t)\}$$

$$= \frac{A}{T} \int_0^\infty t e^{-st} dt$$

using integration by parts

$$\int u\,dv = uv - \int v\,du$$

Let $u = t \rightarrow du = dt$

$e^{-st} dt = dv \rightarrow v = -\frac{1}{s} e^{-st}$

then $\mathcal{L}\{t\, u(t)\} = -\frac{t}{s} e^{-st} \Big|_0^\infty + \int_0^\infty \frac{1}{s} e^{-st} dt$

$$= 0 + \frac{e^{-st}}{s^2} \Big|_0^\infty = \frac{1}{s^2}$$

$$\boxed{\therefore\ F_1(s) = \frac{A}{T} \frac{1}{s^2}}$$

$$\mathcal{L}\{f_2(t)\} = \mathcal{L}\{-\tfrac{A}{T}(t-T) u(t-T)\}$$

$$= -\frac{A}{T} \mathcal{L}\{(t-T) u(t-T)\}$$

$$= -\frac{A}{T} \int_T^\infty (t-T) e^{-st} dt \qquad \text{——— (1)}$$

P 3 (c) (cont'd)

Let $\tau = t - T$ for $t = T,\ \tau = 0$
$t = \infty,\ \tau = \infty$

$d\tau = dt$

$t = \tau + T$

$e^{-st} = e^{-s(\tau+T)} = e^{-s\tau} e^{-sT}$

Eqn (1) becomes

$$\mathcal{L}\{f_2(t)\} = -\frac{A}{T}\int_0^\infty \tau e^{-s\tau} e^{-sT} d\tau$$

$$= -\frac{A}{T}e^{-sT}\int_0^\infty \tau e^{-s\tau} d\tau$$

$$= -\frac{A}{T}e^{-sT}\frac{1}{s^2}$$

$$\boxed{\mathcal{L}\{f_2(t)\} = -\frac{A}{Ts^2}e^{-sT}}$$

$$\mathcal{L}\{f_3(t)\} = \mathcal{L}\{-A\,u(t-T)\}$$

$$= -A\,\mathcal{L}\{u(t-T)\}$$

$$= -A\int_T^\infty e^{-st} dt$$

$$= -A\left[-\frac{1}{s}e^{-st}\Big|_T^\infty\right]$$

$$\boxed{\mathcal{L}\{f_3(t)\} = -A\left[\frac{1}{s}e^{-sT}\right] = -\frac{A}{s}e^{-sT}}$$

(2) + (2) + (2)

$$\boxed{\therefore\ \mathcal{L}\{f(t)\} = \frac{A}{T}\frac{1}{s^2} - \frac{A}{Ts^2}e^{-sT} - \frac{A}{s}e^{-sT}}$$

(P.4) Solve P9-16 in text

9.16(a) $F_1(s) = \dfrac{s+20}{s(s+10)}$

$$= \frac{k_1}{s} + \frac{k_2}{s+10}$$

P.F.E.:

$$k_1 = \left.\frac{s+20}{s+10}\right|_{s=0} = 2$$

$$k_2 = \left.\frac{s+20}{s}\right|_{s=-10} = -1$$

$$\therefore F_1(s) = \frac{2}{s} - \frac{1}{s+10}$$

$$f_1(t) = \mathcal{L}^{-1}\{F_1(s)\}$$

$$\boxed{f_1(t) = \left(2 - e^{-10t}\right)u(t)}$$ 5

P.(4)(b) $\boxed{9\text{-}16(b)}$

$$F_2(s) = \frac{s^2+10s+10}{s(s+10)}$$

$$= \frac{s^2+10s+10}{s^2+10s}$$

From long division:

$$\begin{array}{r} 1 \\ s^2+10s \overline{) s^2+10s+10} \\ \underline{s^2+10s} \\ 10 \end{array}$$

$$F_2(s) = 1 + \frac{10}{s(s+10)}$$

Apply PFE to the 2nd term on the right

$$F_2'(s) = \frac{10}{s(s+10)} = \frac{k_1}{s} + \frac{k_2}{s+10}$$

$$k_1 = \left.\frac{10}{s+10}\right|_{s=0} = 1$$

$$k_2 = \left.\frac{10}{s}\right|_{s=-10} = -1$$

$$\therefore F_2(s) = 1 + \frac{1}{s} - \frac{1}{s+10}$$

$$f_2(t) = \mathcal{L}^{-1}\{F_2(s)\} = \mathcal{L}\left\{1 + \frac{1}{s} - \frac{1}{s+10}\right\}$$

$$\boxed{f_2(t) = \delta(t) + \left(1 - e^{-10t}\right)u(t)}$$

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(P. 5) Solve P 9-26 in text

9-26 (a) $F_1(s) = \dfrac{(s+40)^2}{(s+10)^2(s+100)}$

$$F_1(s) = \frac{1}{s+10}\left[\frac{(s+40)^2}{(s+10)(s+100)}\right] = \frac{1}{s+10}\left[\frac{s^2+80s+1600}{s^2+110s+1000}\right]$$

$$\begin{array}{r} 1 \\ s^2+110s+1000 \overline{)\, s^2+80s+1600} \\ s^2+110s+1000 \\ \hline -30s+600 \end{array}$$

$$= \frac{1}{s+10}\left[1 + \frac{-30s+600}{(s+10)(s+100)}\right]$$

$$= \frac{1}{s+10}\left[1 - \frac{k_1}{s+10} + \frac{k_2}{s+100}\right]$$

$$k_1 = \left.\frac{-30s+600}{s+100}\right|_{s=-10} = \frac{900}{90} = 10$$

$$k_2 = \left.\frac{-30s+600}{s+10}\right|_{s=-100} = \frac{3600}{-90} = -40$$

$$= \frac{1}{s+10}\left[1 + \frac{10}{s+10} - \frac{40}{s+100}\right]$$

$$= \left[\frac{1}{(s+10)} + \frac{10}{(s+10)^2} - \underbrace{\frac{40}{(s+10)(s+100)}}_{F_1'(s)}\right]$$

$$F_1'(s) = \frac{40}{(s+10)(s+100)} = \frac{k_3}{s+10} + \frac{k_4}{s+100}$$

$$k_3 = \left.\frac{40}{s+100}\right|_{s=-10} = \frac{4}{9}$$

$$k_4 = \left.\frac{40}{s+10}\right|_{s=-100} = -\frac{4}{9}$$

$$= \left[\frac{1}{s+10} + \frac{10}{(s+10)^2} - \frac{4}{9}\frac{1}{(s+10)} + \frac{4}{9}\frac{1}{(s+100)}\right]$$

$$F_1(s) = \frac{5}{9}\frac{1}{(s+10)} + \frac{10}{(s+10)^2} + \frac{4}{9}\frac{1}{(s+100)}$$

$$\boxed{f_1(t) = \left[\frac{5}{9}e^{-10t} + 10te^{-10t} + \frac{4}{9}e^{-100t}\right]u(t)}$$

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(P. 5)(a) (cont'd)

P9-26 (b):

$$F_2(s) = \frac{(s+10)^2}{(s+40)^2(s+100)}$$

$$F_2(s) = \frac{1}{(s+40)}\left[\frac{(s+10)^2}{(s+40)(s+100)}\right] = \frac{1}{(s+40)}\left[\frac{s^2+20s+100}{s^2+140s+4000}\right]$$

$$= \frac{1}{(s+40)}\left[1 + \underbrace{\frac{-120s-3900}{(s+40)(s+100)}}_{F_2'(s)}\right]$$

$$\begin{array}{r} 1 \\ s^2+140s+4000 \overline{) s^2+20s+100} \\ s^2+140s+4000 \\ \hline -120s-3900 \end{array}$$

$$= \frac{1}{(s+40)}\left[1 + \frac{15}{(s+40)} - \frac{135}{(s+100)}\right]$$

$$F_2'(s) = \frac{k_1}{s+40} + \frac{k_2}{s+100}$$

$$k_1 = \left.\frac{-120s-3900}{s+100}\right|_{s=-40}$$

$$k_1 = \frac{900}{60} = 15$$

$$k_2 = \left.\frac{-120s-3900}{s+40}\right|_{s=-100}$$

$$k_2 = \frac{8100}{-60} = -135$$

$$= \frac{1}{(s+40)} + \frac{15}{(s+40)^2} - \underbrace{\frac{135}{(s+40)(s+100)}}_{F_2''(s)}$$

$$F_2''(s) = \frac{k_3}{s+40} + \frac{k_4}{s+100}$$

$$k_3 = \left.\frac{135}{s+100}\right|_{s=-40} = \frac{9}{4}$$

$$k_4 = \left.\frac{135}{s+40}\right|_{s=-100} = -9/4$$

$$= \frac{1}{(s+40)} + \frac{15}{(s+40)^2} - \frac{9}{4}\frac{1}{(s+40)} + \frac{9}{4}\frac{1}{(s+100)}$$

$$F_2(s) = -\frac{5}{4}\frac{1}{(s+40)} + \frac{15}{(s+40)^2} + \frac{9}{4}\frac{1}{(s+100)}$$

$$\boxed{f_1(t) = \left[-\frac{5}{4}e^{-40t} + 15te^{-40t} + \frac{9}{4}e^{-100t}\right]u(t)}$$

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>> syms t s
>> F1 = (s+40)^2 / (((s+10)^2) * (s+100) );
>> f1 = ilaplace(F1, s, t);
>> f1_t = simplify( f1 )

f1_t =

4/9*exp(-100*t)+10*exp(-10*t)*t+5/9*exp(-10*t)


>> pretty(f1_t)

              4/9 exp(-100 t) + 10 exp(-10 t) t + 5/9 exp(-10 t)
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>> F2 = (s+10)^2 / (((s+40)^2) * (s+100) );
>> f2 = ilaplace(F2, s, t);

>> f2_t = simplify( f2 )

f2_t =

9/4*exp(-100*t)-5/4*exp(-40*t)+15*exp(-40*t)*t

>> pretty(f2_t)

             9/4 exp(-100 t) - 5/4 exp(-40 t) + 15 exp(-40 t) t
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