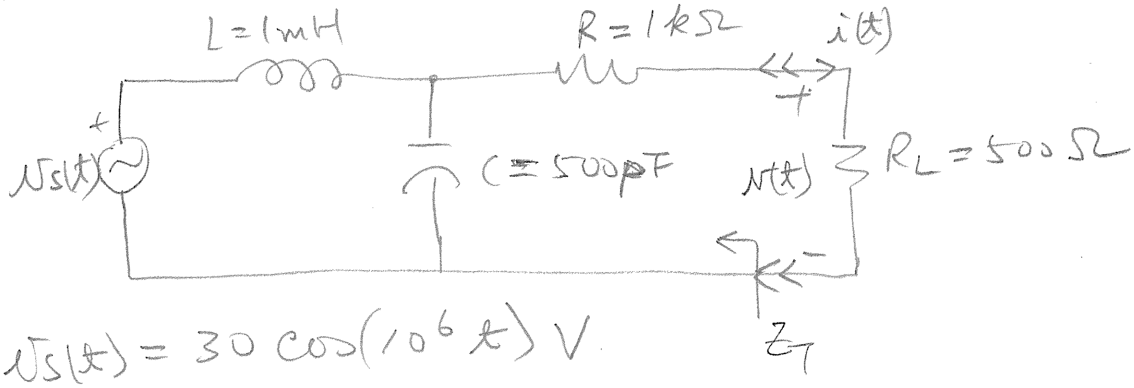
ECE 202 Fall 2007 H.W #5 Solution Key

(P.2) Solve P 8-59 in text



$v_s(t) = 30\cos(10^6 t)$ V

$\therefore\ \omega = 10^6$ rad s^{-1}

obtain the Thévenin equivalent voltage source and impedance:

$$Z_T = 10^3 + \left(\frac{1}{j\omega(0.5\times10^{-9}) + \frac{1}{(j\omega 10^{-3})}}\right)\ \Omega$$

$$Z_T = 1\times10^3 + j2\times10^3 = 1 + j2\ k\Omega$$ (2 pts)

For Thévenin voltage source, in phasor form

$$\vec{V}_T = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L}\cdot \vec{V}_s$$ (2 pts)

$$= \frac{\frac{1}{j(10^6)(0.5\times10^{-9})}}{\frac{1}{j(10^6)(0.5\times10^{-9})} + j(10^6)(10^{-3})}$$

$\vec{V}_T = 60$ V (2 pts)

(P. 2)(P8-59 (a))

For $Z_L = 500\,\Omega$,

$$\vec{I}_L = \frac{\vec{V}_T}{Z_T + 500}$$ (1 pt)

$$P_L = 0.5\,|\vec{I}_L|^2 \cdot 500 \text{ W}$$ (2 pts)

$$\therefore \boxed{P_L = 0.144\text{ W}}$$ (5 pts)

(b) $$P_{MAX} = \frac{|\vec{V}_T|^2}{8\,\mathrm{Re}\{Z_T\}}$$ (1 pt)

$$= \frac{(60)^2}{8(1\times10^3)}$$

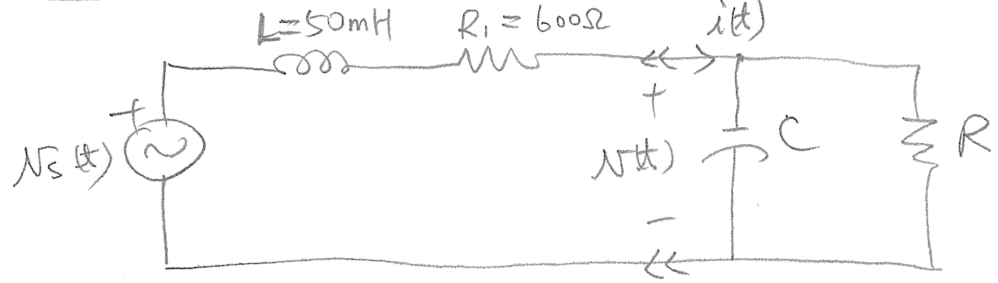
(2 pts) $$\boxed{P_{max} = 0.45\text{ W}}$$ (5 pts)

(c) For max power: $Z_L = Z_T^*$ (1 pt)

thus $$\boxed{Z_L = 1\times10^3 - j2\times10^3\ \Omega}$$ (5 pts)

(2 pts)

(P.3) Solve P8-60 in text.



$v_s(t) = 10\cos(8000t)$ V. $\quad \omega = 8000$ rad s^{-1}

The Thévenin equivalent seen by the load is

(1pt) $\vec{V}_T = 10$ V

$Z_T = R_1 + j\omega L = 600 + j(8000)(50 \times 10^{-3})\ \Omega$

(2pts) $Z_T = 600 + j400\ \Omega$.

(2pts) (a) $P_{max} = \dfrac{|\vec{V}_T|^2}{8\,\mathrm{Re}\{Z_T\}} = \dfrac{(10)^2}{8(600)}$

$$\boxed{P_{max} = 2.083 \times 10^{-2}\ \mathrm{W}}$$ (5pts)

(b) For max average power, the load should be conjugate-matched to the Thévenin impedance:

(2pts) $Z_L = \dfrac{1}{\frac{1}{R} + j\omega C} = Z_T^* = 600 - j400\ \Omega$

$\therefore \dfrac{1}{\frac{1}{R} + j\omega C} = 600 - j400$

(2pts) $\dfrac{1}{R} + j\omega C = \dfrac{1}{600 - j400} = 1.154 \times 10^{-3} + j7.69 \times 10^{-4}$

(P.3)(cont'd)

comparing terms :

$$\frac{1}{R} = 1.154 \times 10^{-3}$$

(2pts) $\therefore$ $\boxed{R = 866.551\ \Omega}$ ($\frac{5pts}{}$)

Also : $j\omega C = j\,7.69 \times 10^{-4}$

$$C = \frac{7.69 \times 10^{-4}}{8000}$$

(2pts) $\boxed{\therefore\ C = 96.15\ nF}$ ($\underline{5pts}$)

(P.4)

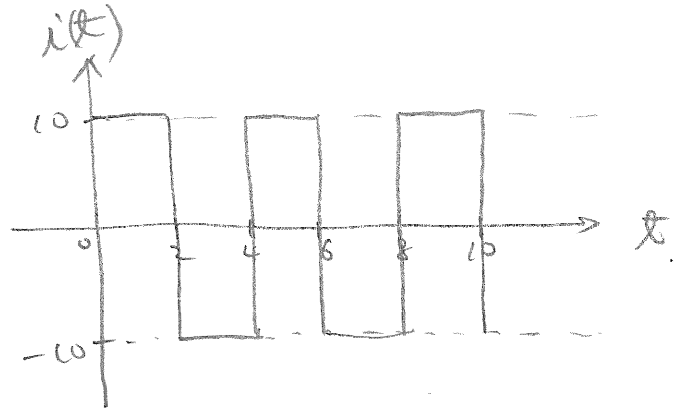


Fig. 1

(a) Find the rms value of $i(t)$ in Fig 1.

For one period, T: $i(t) = \begin{cases} I_m & 0 \le t < 2 \\ -I_m & 2 \le t < 4 \end{cases}$ $\quad I_m = 10$ over 1 period of $T = 4$.

Square: $i^2(t) = \begin{cases} I_m^2 & 0 \le t < 2 \\ I_m^2 & 2 \le t < 4 \end{cases}$ over 1 period T.

2 pts

Mean: mean value $= \frac{1}{T}\int_0^T i^2(t)\,dt$

2 pts

$$= \frac{1}{T}\left\{ \int_0^{T/2} I_m^2\,dt + \int_{T/2}^{T} I_m^2\,dt \right\}$$

$$= \frac{1}{T}\left\{ I_m^2 t\Big|_0^{T/2} + I_m^2 t\Big|_{T/2}^{T} \right\}$$

$$= \frac{1}{T}\left\{ I_m^2 T \right\}$$

2 pts $\therefore$ mean value $= I_m^2$

Root: $\sqrt{I_m^2} \Rightarrow \boxed{I_{rms} = I_m = 10\text{ A}}$ (10 pts)

P.4 (b) The avg pwr dissipated in a 2-Ω resistor is:

(2pts) $P = I_{rms}^2 R = (10)^2 2$ (1pt)

(2pts) $\boxed{P = 200\,W}$ (5 pts)

P.4 (c) For a sinusoidal signal of the same amplitude and frequency (thus, period):

$i(t) = I_m \cos\left[2\pi\left(\frac{1}{T}\right)t\right]$ A.

$i(t) = 10\cos(0.5\pi t)$ A

(2pts) $I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07$ A.

Avg power dissipated in this case is

(2pts) $P = I_{rms}^2 R = (7.07)^2(2)$

(1pt) $\boxed{P = 100\,W}$ (5 pts)

Note: the sinusoidal signal delivers only half the average power of that delivers by the rectangular waveform!!