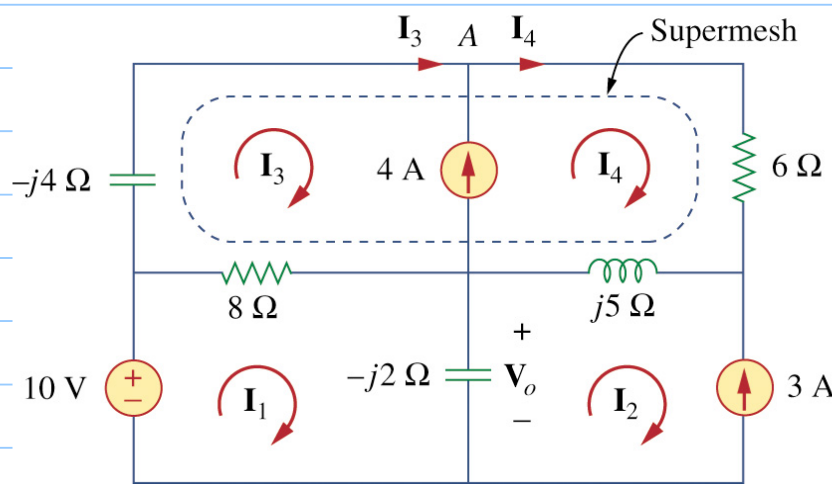
ECE202 (Fall 2007) HW#4 Solutions

(P.1) For the meshes (and supermesh) indicated:



For mesh 1, KVL gives

2 $$-10 + (8 - j2)\vec{I}_1 - (-j2)\vec{I}_2 - 8\vec{I}_3 = 0 \qquad (1)$$

For mesh 2,

2 $$\vec{I}_2 = (-1)\,3\angle 0^\circ = -3 \text{ A} \qquad (2)$$

Meshes 3 & 4 form a supermesh due to the shared current source between them.

For the supermesh:

2 $$(8 - j4)\vec{I}_3 - 8\vec{I}_1 + (6 + j5)\vec{I}_4 - j5\vec{I}_2 = 0 \qquad (3)$$

Also, applying KCL to the node above the top current source yields:

(2) $$\vec{I}_4 = \vec{I}_3 + 4 \qquad (4)$$

(a) Solving by hand

Combining (1) and (2) yields

(2) $$(8-j2)\vec{I}_1 - 8\vec{I}_3 = 10 + j6 \qquad (5)$$

Also, combining (3) and (4) yields

(2) $$-8\vec{I}_1 + (14+j)\vec{I}_3 = -24 - j35 \qquad (6)$$

From equations (5) and (6) we now have the system of equation

(2) $$\begin{bmatrix} (8-j2) & -8 \\ -8 & (14+j) \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 10+j6 \\ -24-j35 \end{bmatrix} \qquad (7)$$

Equation (7) may be solved using Kramer's Rule, or other linear algebra techniques.

Forming the determinants

$$\Delta = \begin{vmatrix} 8-j2 & -8 \\ -8 & 14+j \end{vmatrix} = 112+j8-j28+2-64$$

$$= 50-j20$$

$$\Delta_1 = \begin{vmatrix} 10+j6 & -8 \\ -24-j35 & 14+j \end{vmatrix} = 140+j10+j84-6-192-j280$$

$$= -58-j186$$

The current $\vec{I}_1$ is obtain as:

(2) $$\vec{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58-j186}{50-j20} = 3.618\angle 274.5^\circ \text{ A}$$

The voltage, $\vec{V}_o$, is

$$\vec{V}_o = -j2(\vec{I}_1 - \vec{I}_2) = -j2(3.618\angle 274.5^\circ)$$

$$= -7.2134 - j6.568 \text{ V}$$

(2) $$\boxed{\vec{V}_o = 9.756\angle -137.68^\circ \text{ V}}$$ or

$$\boxed{\vec{V}_o = 9.756\angle 222.32^\circ \text{ V}}$$

(P.1)(b) Using MATLAB.

Equations (1)–(4) form the system of equation

2 $$\begin{bmatrix} (8-j2) & j2 & -8 & 0 \\ 0 & 1 & 0 & 0 \\ -8 & -j5 & (8-j4) & (6+j5) \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \\ \vec{I}_3 \\ \vec{I}_4 \end{bmatrix} = \begin{bmatrix} 10 \\ -3 \\ 0 \\ 4 \end{bmatrix} \quad (8)$$

or $$\underline{A}\,\vec{I} = \vec{b}$$

$\vec{I}$ can be obtained by multiplying both sides by the inverse of A

2 $$\underline{A}^{-1}\,\underline{A}\,\vec{I} = \underline{A}^{-1}\,\vec{b}$$

$$\therefore \quad \vec{I} = \underline{A}^{-1}\,\vec{b}$$

(note: order of multiplication is important!)

Entering the matrix and vector into MATLAB and solving for $\vec{I}$ (and $\vec{V}_s$):

```
>> A = [(8-j*2)   (j*2)      -8         0;
           0          1          0          0;
          -8         (-j*5)   (8-j*4)  (6+j*5);
           0          0          -1         1];
>> b = [10 -3 0 4]';
>> I = A\b

I =
  0.2828 - 3.6069i
 -3.0000 + 0.0000i
 -1.8690 - 4.4276i
  2.1310 - 4.4276i

>> Vo = -2 * j * (I(1) - I(2))

Vo =

 -7.2138 - 6.5655i

>> abs(Vo)

ans =

  9.7542

>> phase(Vo)*180/pi

ans =

 -137.6936
```

°

③ w/ ML code

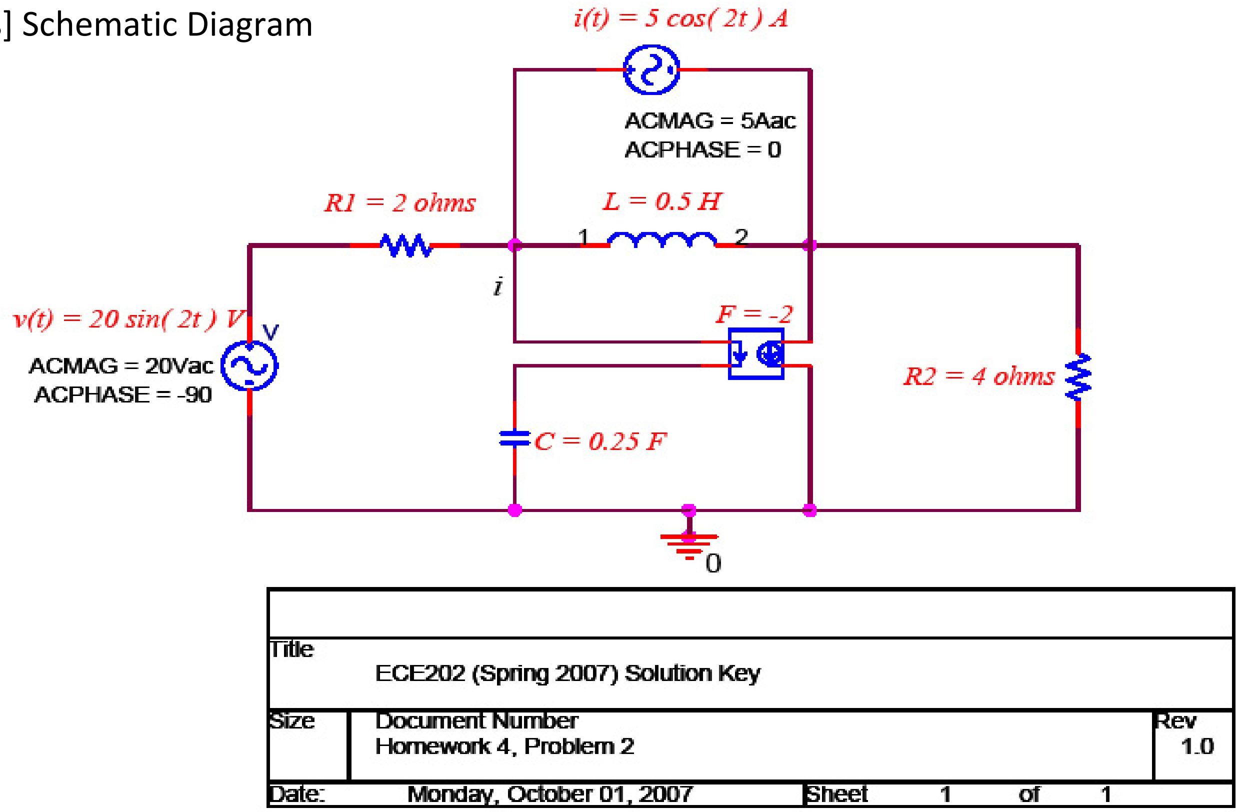
$\therefore \vec{V}_o = 9.7542 \angle -137.69^\circ$ V

(same as in part (a))

HW#4, P.2 [25 pts]

(a) [5 pts] $i(t) = 7.906 \cos(2t - 136.5°)$ A

(b) [15 pts] Schematic Diagram



From simulation output file:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
3.183E-01	7.906E+00	-1.365E+02

(c) [5 pts] SCHEMATIC1.net:

```
* source HW4_P2
R_R1          N09488 N07835  2
R_R2           0 N08350  4
C_C           0 N00532  0.25
L_L           N07835 N08350  0.5
V_V           N09488 0 DC 0Vdc AC 20Vac -90
I_I           N07835 N08350 DC 0Adc AC 5Aac 0
V_PRINT1           N16146 N07835 0V

.PRINT         AC
+ IM(V_PRINT1)
+ IP(V_PRINT1)
X_F1     N16146 N00532 N08350 0 SCHEMATIC1_F1

.subckt SCHEMATIC1_F1 1 2 3 4
F_F1           3 4 VF_F1 -2
VF_F1           1 2 0V
.ends SCHEMATIC1_F1
```