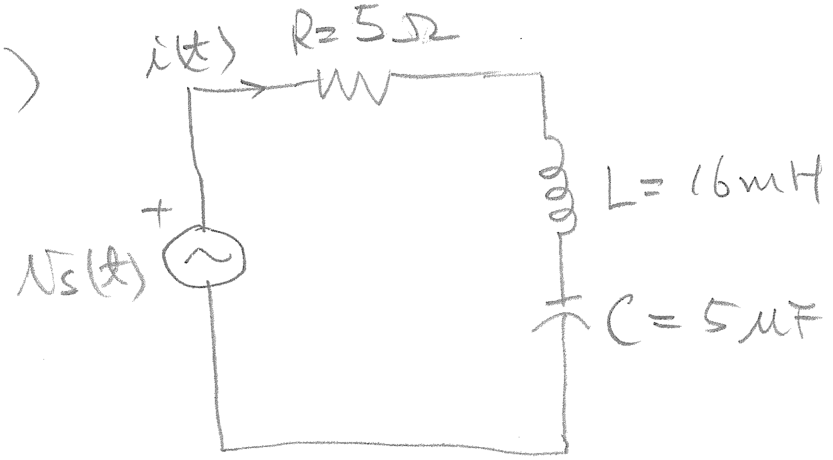
(P.1)

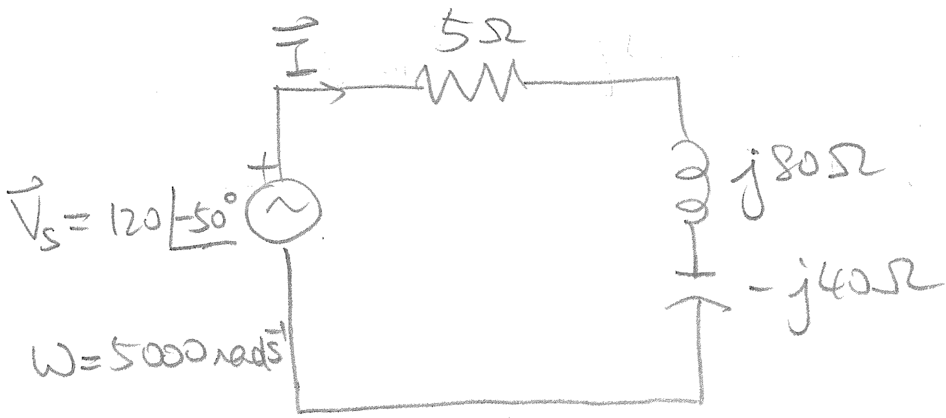


(a) In phasor domain:

$v_s(t) = 120\cos(5000t - 50°) \leftrightarrow \vec{V}_s = 120\angle{-50°}$ V

$Z_L: \quad j\omega L = j(5000)(16\times10^{-3}) = j80\ \Omega$

$Z_C: \quad \dfrac{1}{j\omega C} = \dfrac{1}{j(5000)(5\times10^{-6})} = -j40\ \Omega$



(8 pts)

(b) $Z_{eq} = 5 + j80 - j40 = 5 + j40\ \Omega$ (2 pts)

$= 40.31\angle{82.87°}\ \Omega$

(c) $\vec{I} = \dfrac{\vec{V}_s}{Z_{eq}} = \dfrac{120\angle{-50°}}{40\angle{82.87°}} = 2.977\angle{-132.87°}$ A (2 pts)

(d) $i(t) = 2.977\cos(5000t - 132.87°)$ A (2 pts)

(e) The Equivalent Impedance is

$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C} \qquad (2\text{pts})$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right) = R_{eq} + jX_{eq}$$

At resonance frequency, f_0,
the reactance $X_{eq} = 0$. (2 pts)

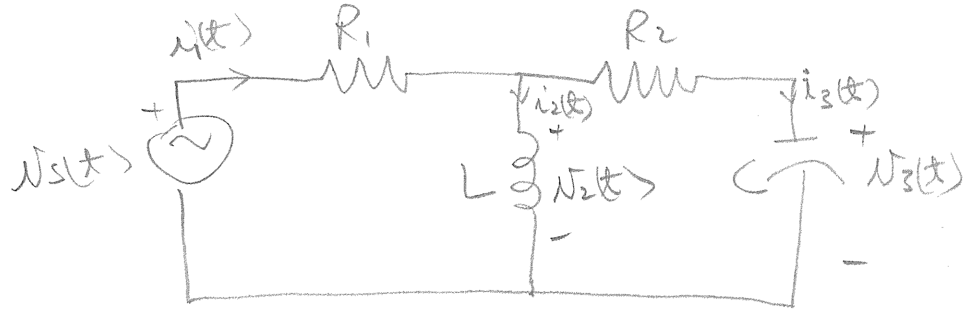
thus. $\omega_0 L - \frac{1}{\omega_0 C} = 0$

and $\omega_0 = \frac{1}{\sqrt{LC}}$

$\therefore \quad \omega_0 = \frac{1}{\sqrt{(16\times10^{-3})(5\times10^{-6})}} = 3535.53 \text{ rads}^{-1}$

$\therefore \quad f_0 = \frac{\omega_0}{2\pi} = \underline{562.70 \text{ Hz}}$ (2pts)

(P2)



$v_s(t) = 24\cos(1000t + 60)$ V

$R_1 = 4\,\Omega$

$R_2 = 8\,\Omega$

$C = 250\,\mu F$

$L = 6\,mH$

Convert to phasors:

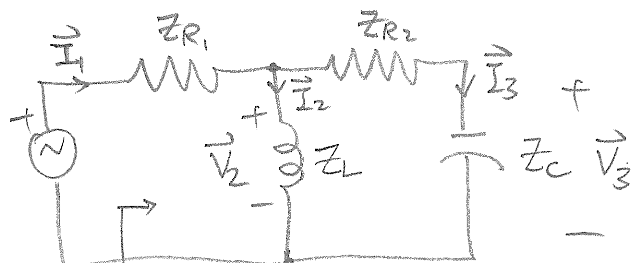
$$\vec{V}_s = 24\angle 60^\circ \text{ V}$$

$$Z_{R1} = 4\ \Omega$$

$$Z_{R2} = 8\ \Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{(1000)(250\times10^{-6})} = -j4\ \Omega$$

$$Z_L = j\omega L = j(1000)(6\times10^{-3}) = j6\ \Omega$$



(2 pts)

$$Z_{eq} = Z_{R_1} + Z_L \,//\, (Z_{R2} + Z_C)$$

(2 pts) $$= Z_{R_1} + \frac{Z_L(Z_{R2}+Z_C)}{Z_L + Z_{R2} + Z_C}$$

$$= 4 + \frac{(j6)(8-j4)}{j6+8-j4} = 4 + \frac{24+j48}{8+j2}$$

(2 pts) $$= 4 + \frac{53.665\angle 63.43^\circ}{8.246\angle 14.04^\circ} = 4 + 6.508\angle 49.40^\circ$$

$$= 4 + 4.235 + j4.941 = 8.235 + j4.941\ \Omega$$

$$\boxed{Z_{eq} = 9.604\angle 30.96^\circ\ \Omega}$$ (2 pts)

$$\vec{I}_1 = \frac{\vec{V}_s}{Z_{eq}} = \frac{24\angle 60^\circ}{9.604\angle 30.96^\circ} = 2.50\angle 29.04^\circ \text{ A}$$

$$\boxed{i_1(t) = 2.50\cos(1000t + 29.04^\circ) \text{ A}}$$ (4pts)

Note that $\vec{V}_2$ may be obtained in two ways: using KVL or voltage division

<u>using KVL</u>

$$\vec{V}_2 = \vec{V}_S - \vec{I}_1 Z_{R_1}$$ (2pts)

$$= 24\angle 60^\circ - 10\angle 29.06^\circ$$

$$= 3.26 + j15.93$$

$$\vec{V}_2 = 16.26\angle 78.43^\circ \text{ V}$$

using voltage division

$$\vec{V}_2 = \vec{V}_s \frac{Z_L \| (Z_{R_2} + Z_c)}{Z_{R_1} + Z_L \| (Z_{R_2} + Z_c)}$$

$$= 24\angle 60^\circ \left(\frac{6.508\angle 49.40^\circ}{9.604\angle 30.96^\circ} \right)$$

$$\vec{V}_2 = 16.26\angle 78.43^\circ$$

$$\boxed{\therefore v_2(t) = 16.26\cos(1000t + 78.43^\circ) \text{ V}}$$ (4pts.)

$$\vec{I}_2 = \frac{\vec{V}_2}{Z_L} = \frac{16.26\angle 78.43^\circ}{6\angle 90^\circ} = 2.71\angle -11.57^\circ \text{ A}$$

$$\boxed{\therefore i_2(t) = 2.71\cos(1000t - 11.57^\circ) \text{ A.}}$$ (4pts.)

$$\vec{I}_3 = \frac{\vec{V}_2}{Z_{R_2} + Z_C} = \frac{16.26\angle 78.43^\circ}{8 - j4} = \frac{16.26\angle 78.43^\circ}{8.944\angle -26.57^\circ}$$

$$= 1.818\angle 105^\circ \text{ A}$$

$$\boxed{i_3(t) = 1.818\cos(1000t + 105^\circ) \text{ A.}}$$ (4pts.)

Note that $\vec{I}_2$ & $\vec{I}_3$ could have been obtained using current division.

Finally,

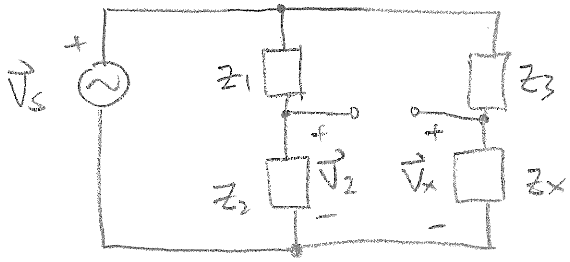
$$\vec{V}_3 = \vec{I}_3 Z_C = 1.818 \angle 105^\circ (-j4)$$

$$= 1.818 \angle 105^\circ \left(4 \angle -90^\circ\right)$$

$$\vec{V}_3 = 7.27 \angle 15^\circ \text{ V}$$

$$\boxed{\therefore\ v_3(t) = 7.27 \cos(1000t + 15^\circ) \text{ V}} \quad (4\text{ pts})$$

P3.



$Z_1 = 1\,k\Omega$

$Z_2 = 4.2\,k\Omega$

$f = 2\,kHz$

$\omega = 2\pi f = 4000\pi \text{ rad s}^{-1}$

For a balanced bridge, the current through the ac meter is zero ⟶ can replace the meter with an open circuit.

$$\vec{V}_2 = \vec{V}_s \frac{Z_2}{Z_1 + Z_2}$$ the 1st branch (2 pts)

And $$\vec{V}_x = \vec{V}_s \frac{Z_x}{Z_x + Z_3}$$ the 2nd branch (2 pts)

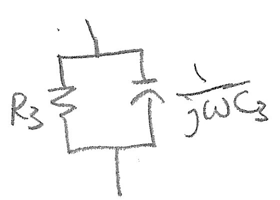
Since $$\vec{V}_2 = \vec{V}_x$$ (for balanced bridge)

then $$\frac{Z_2}{Z_1 + Z_2} = \frac{Z_x}{Z_x + Z_3}$$

$$Z_2 Z_x + Z_2 Z_3 = Z_x Z_1 + Z_x Z_2$$

$$Z_2 Z_3 = Z_x (Z_1 + Z_2 - Z_2)$$

$$\therefore \quad Z_x = \frac{Z_2 Z_3}{Z_1} \qquad \text{——— (1) (2 pts)}$$

$Z_3 =$ 

$= R_3 \,||\, \frac{1}{j\omega C_3} = \dfrac{\frac{R_3}{j\omega C_3}}{R_3 + \frac{1}{j\omega C_3}}$

$$Z_3 = \frac{R_3}{1 + j\omega R_3 C_3} \qquad (2\text{ pts})$$

P3 (cont'd)

Given: $R_3 = 1.5\ M\Omega$
$C_3 = 12\ pF$

Then

$$Z_3 = \frac{1.5\times10^6}{1 + j(4000\pi)(1.5\times10^6)(12\times10^{-12})}\ \Omega$$

$$= \frac{1.5\times10^6}{1 + j0.2262} = \frac{1.5\times10^6 \angle 0^\circ}{1.025 \angle 12.746^\circ}$$

$$= 1.463 \angle -12.746^\circ\ M\Omega$$

$$Z_3 = 1.427 - j0.3228\ M\Omega \quad (2\text{ pts})$$

sub into (1) we have

$$Z_x = \frac{(4.2\times10^3)}{(1\times10^3)}(1.463\angle -12.746^\circ)\ M\Omega$$

$$Z_x = 6.145 \angle -12.746^\circ\ M\Omega$$

$$Z_x = 5.993 - j1.356\ M\Omega \quad (2\text{ pts})$$

P.3(a) For a series circuit implementation:

Let $Z_x = R_x + jX_x = 5.993 - j1.356\ M\Omega$

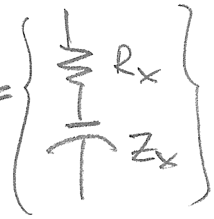
Since X_x is negative, we choose a capacitor

$$\therefore Z_x = R_x + \frac{-j}{\omega C_x} = 5.993 - j1.356\ M\Omega \quad (2\text{ pts})$$

$$\therefore \boxed{R_x = 5.993\ M\Omega} \quad (2\text{ pts})$$

↓

P-3(a) cont'd

$Z_x = $  (1pt)

$$X_x = \frac{1}{\omega C_x} = 1.356\ M$$

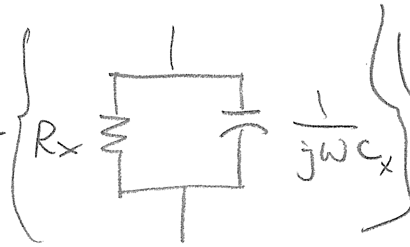
$$\therefore \quad C_x = \frac{1}{(1.356 \times 10^6)(4000\pi)}$$

$$\boxed{C_x = 58.69\ pF}$$ (2pts)

P3(b) For a parallel circuit implementation:

Let $Z_x = (5.993 - j1.356)\ M\Omega$

$$= R_x // \frac{1}{j\omega C_x}$$

$Z_x = $  (1pt)

$$Z_x = \frac{R_x}{1 + j\omega R_x C_x}$$ (2pts)

$$= \frac{R_x}{1 + j\omega R_x C_x} \cdot \frac{1 - j\omega R_x C_x}{1 - j\omega R_x C_x} = \frac{R_x(1 - j\omega R_x C_x)}{1 + \omega^2 R_x^2 C_x^2}$$

$$= \frac{R_x}{1 + \omega^2 R_x^2 C_x^2} - j\frac{\omega R_x^2 C_x}{1 + \omega^2 R_x^2 C_x^2}$$ (2pts)

Comparing terms we have

$$\frac{R_x}{1 + \omega^2 R_x^2 C_x^2} = 5.993 \times 10^6 \quad \text{—(2)}$$ (1 pt)

and

$$\frac{\omega R_x^2 C_x}{1 + \omega^2 R_x^2 C_x^2} = 1.356 \times 10^6 \quad \text{—(3)}$$ (1 pt)

↓

P3(b) (Cont'd)

Dividing (3) by (2) yields:

$$\frac{Eq(3)}{Eq(2)} = \frac{\omega R_x^2 C_x}{R_x} = \omega R_x C_x = \frac{1.356\times10^6}{5.993\times10^6}$$

$$\therefore (4000\pi) R_x C_x = 0.22626$$

$$R_x C_x = 1.80\times10^{-5}$$

$$\therefore R_x = \frac{1.80\times10^{-5}}{C_x} \quad \text{—— (4)}$$

Sub (4) into (2) yields.

$$\frac{\frac{1.80\times10^{-5}}{C_x}}{1+(4000\pi)^2\left(\frac{1.80\times10^{-5}}{C_x}\right)^2 C_x^2} = 5.993\times10^6$$

$$\frac{1.80\times10^{-5}}{C_x}\cdot\frac{1}{1.05} = 5.993\times10^6$$

$$\therefore C_x = \frac{1.80\times10^{-5}}{(1.05)(5.993\times10^6)}$$

$$\boxed{\therefore C_x = 2.86 \text{ pF}}$$ (2 pts)

Sub C_x into (4) yields.

$$R_x = \frac{1.80\times10^{-5}}{2.86\times10^{-12}}\ \Omega$$

$$\boxed{\therefore R_x = 6.30\ \text{M}\Omega}$$ (2pts)