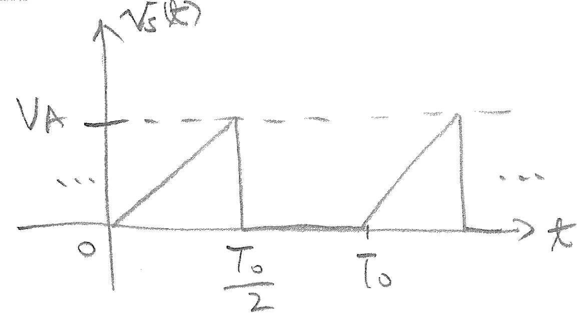
ECE202 Fall 2007 H.W # 10
Solutions.

P.1 (Problem 13-4 in text)

[16]



$$v(t) = \begin{cases} V_A \dfrac{2}{T_0} t & 0 \le t < \dfrac{T_0}{2} \\ 0 & \dfrac{T_0}{2} < t < T_0 \end{cases}$$

$$a_0 = \frac{1}{T_0}\int_0^{\frac{T_0}{2}} \frac{2V_A}{T_0} t\,dt = \frac{2V_A}{T_0^2}\frac{t^2}{2}\Big|_0^{\frac{T_0}{2}} = \frac{V_A}{T_0^2}\left[\frac{T_0^2}{4}\right]$$

$$\therefore \boxed{a_0 = \frac{V_A}{4}} \quad (4)$$

$x = t$

$a = \dfrac{2\pi n}{T_0}$

$$a_n = \frac{2}{T_0}\int_0^{\frac{T_0}{2}} \frac{2V_A}{T_0} t \cos\left(\frac{2\pi n}{T_0}t\right)dt$$

since $\displaystyle\int x\cos(ax)\,dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax)$

$$\therefore a_n = \frac{4V_A}{T_0^2}\left[\frac{T_0^2}{2^2\pi^2 n^2}\cos\left(\frac{2\pi n}{T_0}t\right) + \frac{t\,T_0}{2\pi n}\sin\left(\frac{2\pi n}{T_0}t\right)\right]\Bigg|_0^{\frac{T_0}{2}}$$

$$= \frac{4V_A}{T_0^2}\left\{\frac{T_0^2}{4(\pi n)^2}\left[\cos\left(\frac{2\pi n}{T_0}\frac{T_0}{2}\right) - \cos(0)\right]\right.$$

$$\left. + \left[\frac{\frac{T_0}{2}(T_0)}{2\pi n}\sin\left(\frac{2\pi n}{T_0}\frac{T_0}{2}\right) - 0\right]\right\}$$

$$= \frac{4V_A}{T_0^2}\left\{\frac{T_0^2}{4(n\pi)^2}\left[\cos(n\pi)-1\right] + \cancel{\frac{T_0^2}{4\pi n}\sin(n\pi)}^{\,0}\right\}$$

$$= \begin{cases} \frac{V_A}{(n\pi)^2}(-2) & \text{for } n \text{ odd} \\ 0 & \text{for } n \text{ even} \end{cases}$$

$$\boxed{\therefore\ a_n = \begin{cases} \frac{-2V_A}{(n\pi)^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}}$$

4 2

$$b_n = \frac{2}{T_0}\int_0^{T_0/2} \frac{2V_A}{T_0}\, t \sin\left(\frac{2\pi n}{T_0}t\right) dt \qquad \begin{array}{l} x = t \\ a = \frac{2\pi n}{T_0} \end{array}$$

Recall: $\int x\sin(ax)\,dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax)$

$$b_n = \frac{4V_A}{T_0^2}\left[\frac{T_0^2}{4(n\pi)^2}\sin\left(\frac{2\pi n}{T_0}\right)t - \frac{t\,T_0}{2\pi n}\cos\left(\frac{2\pi n}{T_0}\right)t\right]\Bigg|_0^{T_0/2}$$

$$= \frac{4V_A}{T_0^2}\left\{\frac{T_0^2}{4(n\pi)^2}\left[\sin\left(\frac{2\pi n}{T_0}\frac{T_0}{2}\right)-0\right] - \left[\frac{T_0^2}{4n\pi}\cos\left(\frac{2\pi n}{T_0}\frac{T_0}{2}\right)-0\right]\right\}$$

$$= \frac{4V_A}{T_0^2}\left\{0 - \frac{T_0^2}{4n\pi}\cos(n\pi)\right\} = -\frac{V_A}{n\pi}\cos(n\pi)$$

$$= \frac{V_A}{n\pi}(-1)\cdot(-1)^n = \frac{V_A}{n\pi}(-1)^{n+1}$$

$$\therefore\ \boxed{b_n = \frac{V_A}{n\pi}(-1)^{n+1} \quad \forall n}$$

4 2

P.2 (Problem 13-13 in text)

[29] The first terms of a Fourier series are:

$$v(t) = 15\left[\sin(20\pi t) - \frac{1}{9}\sin(60\pi t) + \frac{1}{25}\sin(100\pi t) - \frac{1}{49}\sin(140\pi t) + \dots\right]$$

2(a)
[9]

$$\boxed{\begin{aligned} &② \; f_0 = 10 \text{ Hz} \\ &② \; \omega_0 = 2\pi f_0 = 62.832 \text{ rads}^{-1} \\ &② \; T_0 = \frac{1}{f_0} = \frac{1}{10} = 0.1 \text{ sec} \\ &\text{Harmonics are } \underset{①}{3^{rd}},\ \underset{①}{5^{th}} \text{ and } \underset{①}{7^{th}} \end{aligned}}$$

2b
[4]

Since only odd harmonics are present, the waveform has <u>odd</u> ② & <u>half-wave</u> ② symmetry

2c
[16]

let $g(t) = v\left(t - \frac{T_0}{4}\right)$,

then $g(t) = 15\left[\sin\left(20\pi t - \frac{\pi}{2}\right) - \frac{1}{9}\sin\left(60\pi t - 3\frac{\pi}{2}\right) + \frac{1}{25}\sin\left(100\pi t - 5\frac{\pi}{2}\right) - \frac{1}{49}\sin\left(140\pi t - 7\frac{\pi}{2}\right) + \dots\right]$

Since $\sin\left(x - n\frac{\pi}{2}\right) = \sin x \cos\left(n\frac{\pi}{2}\right) - \cos(x)\sin\left(\frac{n\pi}{2}\right)$

$$= -\sin\left(n\frac{\pi}{2}\right)\cdot\cos(x)$$

therefore,

$n=1$ $\quad \sin(20\pi t - \frac{\pi}{2}) = \left[-\cancel{\sin\left(\frac{\pi}{2}\right)}^{1} \cos(20\pi t)\right] \cdot 1$

$n=3$ $\quad -\frac{1}{9} \sin(60\pi t - 3\frac{\pi}{2}) = \left[-\cancel{\sin\left(3\frac{\pi}{2}\right)}^{-1} \cos(60\pi t)\right] \cdot \left(-\frac{1}{9}\right)$

$n=5$ $\quad \frac{1}{25} \sin(100\pi t - 5\frac{\pi}{2}) = \left[-\cancel{\sin\left(5\frac{\pi}{2}\right)}^{1} \cos(100\pi t)\right] \cdot \frac{1}{25}$

$n=7$ $\quad -\frac{1}{49} \sin(140\pi t - 7\frac{\pi}{2}) = \left[-\cancel{\sin\left(7\frac{\pi}{2}\right)}^{-1} \cos(140\pi t)\right] \left(-\frac{1}{49}\right)$

$$= \begin{cases} -\cos(20\pi t) \\ -\frac{1}{9}\cos(60\pi t) \\ -\frac{1}{25}\cos(100\pi t) \\ -\frac{1}{49}\cos(140\pi t) \end{cases}$$

2c

$\therefore$

[16]

$$\boxed{g(t) = 15\left[-\cos(20\pi t) - \frac{1}{9}\cos(60\pi t) - \frac{1}{25}\cos(100\pi t) - \frac{1}{49}\cos(140\pi t) + \ldots \right]}$$

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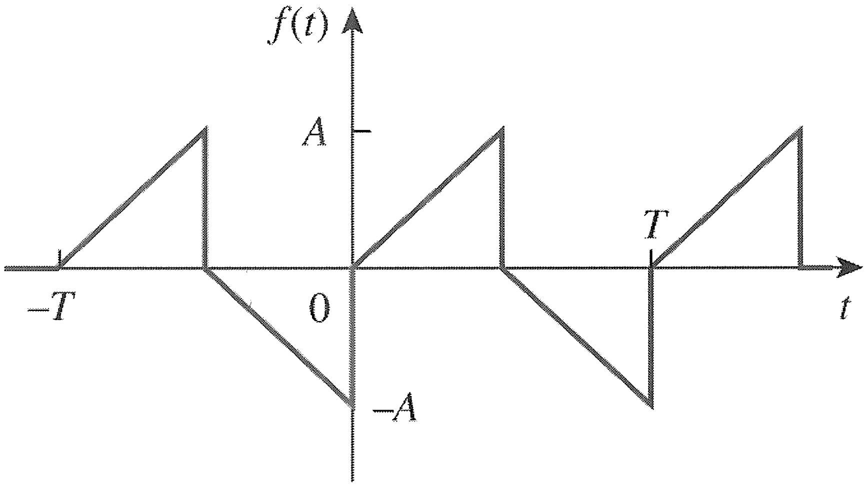
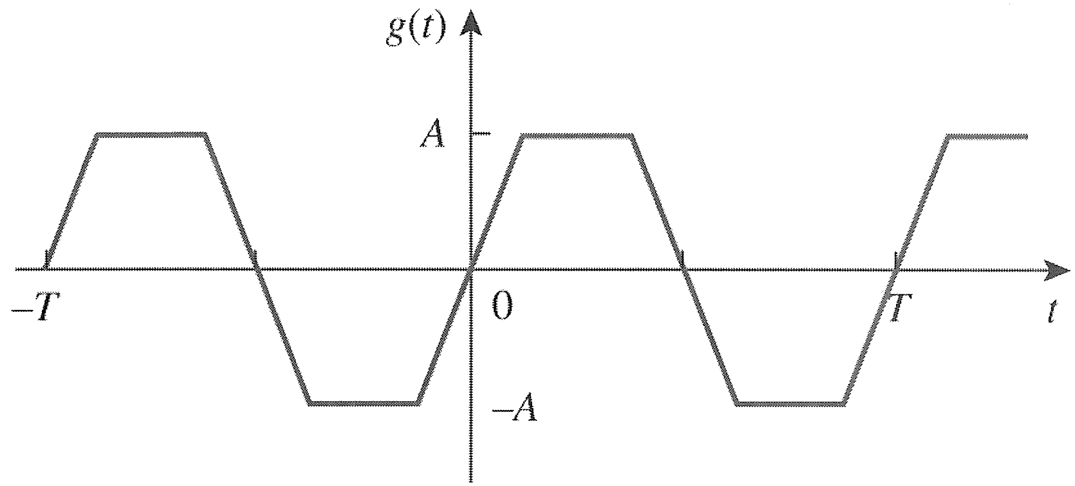
P.3
[6]



(a)

- Half-wave symmetric ②



(b)

- Half-wave symmetric ②
- Odd symmetric ②

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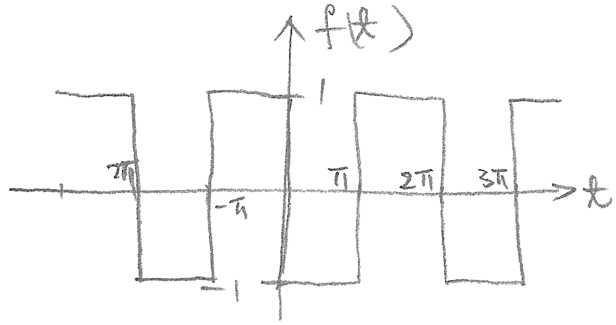
P.4
[19]



(i)
[2]

$\boxed{f(t) \text{ is } \underline{\text{odd}} \text{ and } \underline{\text{half-wave}} \text{ symmetric}}$

1 1

(ii) Since f(t) is odd and half-wave symmetric,
[17]

$$\boxed{\begin{aligned} &2 \quad a_0 = 0 \\ &4 \quad a_n = 0 \quad \forall n. \end{aligned}}$$

$T = 2\pi \qquad \omega_0 = \frac{2\pi}{T} = 1$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_0 t)\,dt$$

$$= \frac{4}{2\pi}\int_0^{\pi} (-1)\sin(nt)\,dt$$

$$= \frac{4}{2\pi}\left(\frac{1}{n}\right)\Big[\cos nt\Big]_0^{\pi} = \frac{2}{n\pi}\big[\cos(n\pi) - 1\big]$$

$$= \begin{cases} \dfrac{-4}{n\pi} & \text{for } n \text{ odd} \\ 0 & \text{for } n \text{ even} \end{cases}$$

4 4 3

$$\therefore \boxed{f(t) = \frac{-4}{\pi}\sum_{k=1}^{\infty} \frac{1}{n}\sin nt \qquad n = 2k-1}$$

$$\text{or } \boxed{f(t) = \frac{-4}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1}\sin[(2n-1)t]}$$

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