ECE 202 Supplement to Sec 8-6 (MSC)

Effective or RMS Values

- The idea of effective value arises from the need to measure the effectiveness of a voltage or current source in delivering power to a resistive load (recall: Avg{sin} = 0)

- The effective value of a periodic signal is its root-mean-square (RMS) value

- The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current (similar for effective voltage)
  - see Sec E.4 in e-Notes Ch 8 (Coffee pot – car batteries example)

Average Power delivered/dissipated in a resistor, R, is

(General periodic excitation)
$$\boxed{P_R = I_{rms}^2 R \quad W}$$

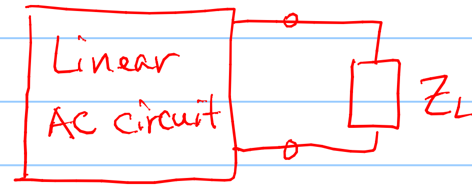
For a sinusoidal excitation current
$$i(t) = I_m \cos(\omega t + \phi) \quad A$$

(Sinusoidal Excitation)
$$P_R = \frac{1}{2} I_m^2 R \quad W$$

Maximum Average Power Transfer

Problem:

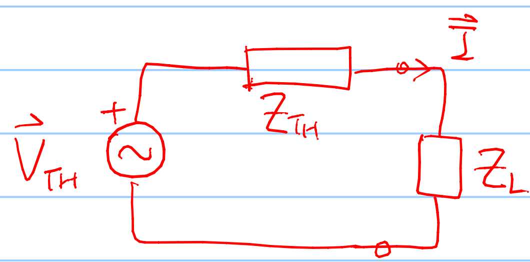
For an ac circuit:



(a) What is Z_L for maximum average power transfer?

(b) What is the maximum average power delivered to the load?

Soln: (a) Represent the linear ac circuit by its Thévenin equivalent:



In rectangular form:

$$Z_{TH} = R_{TH} + jX_{TH} \quad (1)$$

$$Z_L = R_L + jX_L \quad (2)$$

The current, $\vec{I}$, through the load is:

$$\vec{I} = \frac{\vec{V}_{TH}}{Z_{TH} + Z_L} = \frac{\vec{V}_{TH}}{(R_{TH} + jX_{TH}) + (R_L + jX_L)} \quad (3)$$

Recall, the average power delivered to the load is:

$$P = \tfrac{1}{2} |\vec{I}|^2 R_L \quad (4)$$

Substituting (3) into (4) yields:

$$P = \frac{1}{2} R_L \frac{|\vec{V}_{TH}|^2}{(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2} \quad (5)$$

For maximum P w.r.t. R_L and X_L, set

$$\frac{\partial P}{\partial X_L} = 0 \text{ and } \frac{\partial P}{\partial R_L} = 0$$

From (5), we have

$$\frac{\partial P}{\partial X_L} = -|\vec{V}_{TH}|^2 R_L \frac{(X_{TH} + X_L)}{[(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2]^2} \quad (6)$$

and

$$\frac{\partial P}{\partial R_L} = |\vec{V}_{TH}|^2 \frac{[(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2 - 2R_L(R_{TH} + R_L)]}{2[(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2]^2} \quad (7)$$

Setting (6) to zero gives:

$$X_L = -X_{TH} \qquad (8)$$

and setting (7) to zero gives:

$$R_L = \sqrt{R_{TH}^2 + (X_{TH} + X_L)^2} \qquad (9)$$

Combining (8) and (9) leads to the conclusion that, for maximum average transfer, Z_L must be selected such that

$$Z_L = R_L + jX_L \qquad (10)$$

where

$$R_L = R_{TH} \qquad (11)$$

and

$$X_L = -X_{TH} \qquad (12)$$

Thus, for max. avg. pwr. transfer in sinusoidal steady state,

$$\boxed{\begin{aligned} Z_L &= R_L + jX_L = R_{TH} - jX_{TH} \\ Z_L &= Z_{TH}^* \end{aligned}} \qquad (13)$$

(∴ Load = complex conjugate of the Thévenin impedance Z_{TH}
= conjugate-matched to Z_{TH})

(b) Substituting (11) and (12) into (5) gives the maximum average power
(i) delivered to the load
(ii) absorbed by the load
as:

$$\boxed{P_{max} = \frac{|\vec{V}_{TH}|^2}{8 R_{TH}} \ (W)} \qquad (14)$$

Note:

- Conjugate matching applies only when the source is fixed and the load is adjustable (e.g., in power-limited communication systems).
- Conjugate matching does not apply to electrical power systems due to different power transfer constraints. (See Ch 16)
- See Example 8-28 in text
- Work Exercise 8-20 in text