Network Stability.

- A circuit is STABLE if its impulse response $h(t)$ is bounded (i.e $h(t)$ converges to a finite value) as $t \to \infty$.
- It is UNSTABLE if $h(t)$ grows w/out bound as $t \to \infty$.

Mathematically,

$$\lim_{t \to \infty} |h(t)| = \text{finite} \qquad (1)$$

Since $H(s)$ is the Laplace transform of the impulse response $h(t)$, $H(s)$ must meet certain requirements for $h(t)$ to be stable

$$H(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s+p_1)(s+p_2)\ldots(s+p_n)}$$

$H(s)$ must meet 2 requirements for the circuit to be stable :

(I) the degree of $N(s)$ must be less than that of $D(s)$.
Otherwise, long division would yield:

$$H(s) = k_n s^n + k_{n-1} s^{n-1} + \dots + k_1 s + k_0 + \frac{R(s)}{D(s)} \quad (2)$$

where the degree of the remainder, $R(s)$, is less than that of $D(s)$.

* The ILT of (2) does not meet the condition in (1)

(II) All poles of $H(s)$ must have $-$ve real parts (i.e. all poles must be in the Left half of the complex plane so that the ILT would yield:

$$h(t) = \left(k_1 e^{-p_1 t} + k_2 e^{-p_2 t} + \dots + k_n e^{-p_n t}\right) u(t) \quad (3)$$

From (3), all poles must be $+$ve (i.e. pole $s = -p_i$ in the LHP) in order for $e^{-p_i t}$ to decrease as $t \to \infty$!