A Multistep Linear Prediction Approach to Blind Asynchronous CDMA Channel Estimation and Equalization

Jitendra K. Tugnait, Fellow, IEEE, and Tong-tong Li, Student Member, IEEE

Abstract—A multistep linear prediction approach is presented for blind channel estimation, multiuser interference (MUI) suppression, and detection of asynchronous short-code direct sequence code division multiple access signals in multipath channels. Only the spreading code of the desired user is assumed to be known; its transmission delay may be unknown. We exploit the recently proposed multistep linear prediction approach for blind multiple-input multiple-output channel estimation in conjunction with the structure imposed by the desired user’s spreading code sequence. With the knowledge of the desired user’s code sequence, only the second-order statistics of the data are needed under certain sufficient conditions on the underlying multiuser MIMO transfer function. Based on the desired user’s channel estimate, a linear minimum mean square error filter is designed for simultaneous equalization and MUI suppression. Three illustrative simulation examples are presented.

Index Terms—Blind estimation, DS-CDMA, ISI channels, multiaccess communication, multistep linear prediction, multiuser detection, multiuser interference, spread spectrum communication.

I. INTRODUCTION

DIRECT sequence code division multiple access (DS-CDMA) systems have been a subject of intense research interest in recent years. In CDMA systems, multiple users transmit signals simultaneously leading to multiuser interference (MUI). In addition to MUI, presence of multipath propagation introduces intersymbol interference (ISI) causing distortion of the spreading code sequences. Moreover, in reverse links, unknown transmission delays (user asynchronism) also contribute to performance degradation. In future/planned high-rate CDMA systems, the processing gain (chips/symbol) can be much lower (as low as the order of about ten [18]) than that for low-rate voice applications due to bandwidth constraints and multicode approaches [11], [18]. Finally, unlike in low-rate systems, ISI can be significant (due to multipath delays of the order of several symbol periods) in high-rate CDMA systems.

There are two main approaches to the CDMA signal detection problem [16]. The conventional DS-CDMA detector follows a single-user detection strategy where the interfering users are modeled as noise [16], [17]. The RAKE receiver and the matched filter are examples of this strategy [16], [17]. Such receivers are sensitive to the near–far problem and have limited performance in multipath channels [6], [19]. Significant improvement can be obtained with multiuser detectors where the MUI is explicitly part of the signal model [1], [2], [5]–[10], [12]–[14], [16], [19]. Linear multiuser detectors offer an attractive performance-to-complexity tradeoff and have received significant attention in the literature [14]. Implementation of these receivers requires the knowledge of the spreading code, timing (bit/symbol epoch and carrier phase), and channel impulse response for the desired user, and possibly for MUIs. While such information can be acquired by using pilot (training) signals, blind methods which offer better spectrum efficiency by not requiring pilot signals have received increasing attention [1], [2], [5]–[10], [12]–[14], [19]. This paper is concerned with blind detection of the desired user signal for DS-CDMA systems.

Blind methods have typically been used with short spreading codes where the codes repeat every information symbol [7], [14]. This is in contrast with the conventional DS-CDMA detector where use of aperiodic (long) spreading codes extending over a large number of symbols is common (e.g., IS-95 standard). A linearly modulated digital communications signal is a scalar cyclostationary process with period equal to the symbol period. DS-CDMA signals with short spreading codes fall into this category. After chip-rate sampling, the aforementioned signal can be modeled as a vector stationary process [5]. For systems with long (known) codes, the chip-rate sampled signal and MUIs can only be modeled as time-varying scalar cyclostationary processes or time-varying vector processes. The time-varying nature of the received signal models in the case of long codes severely complicates the development of blind approaches as consistent estimation of the needed signal statistics cannot be achieved by time-averaging over the received data record. In contrast, for short codes, one has time-invariant MIMO signal models (see Section II) which allows consistent estimation of the needed signal statistics. This paper is restricted to short spreading codes, as in [1], [2], [5]–[10], [12]–[14] and [19].

In this paper, we consider blind detection (i.e., no training sequence) of the desired user signal, given knowledge of its spreading code, in the presence of MUI, ISI, and user asynchronism (lack of knowledge of user transmission delays, including that of the desired user). Past work on blind detection


This work was supported by the National Science Foundation under Grant CCR-9803850 and by the Office of Naval Research under Grant N00014-97-1-0822.

J. K. Tugnait is with the Department of Electrical and Computer Engineering, Auburn University, Auburn, AL 36849 USA.

T. Li was with the Department of Electrical and Computer Engineering, Auburn University, Auburn, AL. She is now with Bell-Labs, Lucent Technologies, Holmdel, NJ 07733 USA.

Publisher Item Identifier S 0733-8716/01/01905-9.
of DS-CDMA signals includes [1], [2], [5]–[10], [13], [14], and [19], and references therein. In [1] an inverse filtering approach (direct equalizer design) using the second-order statistics, knowledge of the desired user’s code, and of the desired user’s transmission delay, has been presented. It is an extension of [14] to include multipaths. A more general approach is given in [8] under the same assumptions as in [1]. The approaches of [1] and [8] are confined to high SNRs and small multipath delays (a fraction of the symbol period). Subspace-based approaches exploiting the desired user’s spreading code structure have been proposed by several investigators [2], [5], [6], [10], [14], [19]. The methods of [2], [5], [10], and [14] are concerned with blind channel estimation, whereas [6] and [19] directly design a minimum mean square error (MMSE) equalizer. In [9], constant modulus algorithm (CMA) has been used where an exhaustive search for the desired user’s signal has been carried out.

In this paper, we investigate the application of the recently proposed multistep linear prediction (MSLP) approach [3], [4] for blind multiple-input multiple-output (MIMO) channel estimation and equalization to the problem of blind channel estimation and detection of a desired asynchronous DS-CDMA signal. First, we apply the approach of [3] and [4] to characterize the MIMO channel impulse response in Sections III-A and III-B. Then, in Sections III-C and III-D, we exploit the structure imposed by the desired user’s spreading code sequence to discriminate between the desired user’s signal and the MUI. Others ([1], [2], [5]–[10], [12], [19]) have used such a structure. The basic difference between the prior work [3] and [4] and this paper is that [3] and [4] deal with general MIMO models where estimation of the MIMO channel cannot be accomplished without using higher order statistics of the data. This paper deals with DS-CDMA signals where the known spreading code of the desired user allows one to estimate the desired user’s channel by using only the second-order statistics of the data. In [5], it is required that a certain channel matrix (a Sylvester matrix corresponding to the MIMO channel impulse response) should have full column rank for an appropriate choice of a smoothing factor. In [2] also, such a condition has been assumed to define the signal subspace (although in a “deterministic” context). If this condition is not satisfied, the approaches of [2] and [5] may not necessarily work because, then, the range space of a data correlation matrix (as in [5]), or of a data matrix (as in [2]), does not equal the signal subspace. This condition is equivalent to the assumption that the MIMO transfer function $\mathcal{H}(z)$ [see (16)] of the underlying system model is irreducible as well as column reduced [4], [12]. The same assumption has also been made either implicitly or explicitly in [1], [6], [8], [10], and [19]. In this paper, we do not require the MIMO transfer function $\mathcal{H}(z)$ to be column reduced. It has been shown in [12] that even for a forward link of a CDMA system where we have synchronous transmission, $\mathcal{H}(z)$ need not be column-reduced. In [12] a (single-step) linear prediction (SSLP) approach has been advocated. In this paper, motivated by the results of [3] and [4] where the MSLP approach has been shown to significantly outperform the SSLP approach, we propose to use the MSLP approach. Also, unlike [12], we consider reverse link where the transmitted signals are asynchronous. Moreover, unlike [1], [2], [5], [7], [8], and [12], we do not assume synchronization with the desired user’s channel. (We do note that [1], [2], [5], and [7] propose searching for the desired user’s transmission delay—we do not necessarily need such a search.)

The underlying system model is discussed in Section II. The MSLP based solution is presented and analyzed in Section III. Three illustrative simulation examples are provided in Section IV where we compare the performance of the proposed approach to that of [2], [5], [6], and [19].

II. System Model

Consider an asynchronous short-code DS-CDMA system with $M$ users and $N$ chips per symbol with the $j$th user’s spreading code denoted by $\mathbf{c}_j = [c_j(0), \ldots, c_j(N-1)]^T$. Then, the $j$th user’s transmitted signal at the chip rate in a baseband discrete-time model representation is given by [1], [2], [5]–[10], [19]

$$x_j(n) = \sum_{k=-\infty}^{\infty} s_j(k)c_j(n-kN), \quad j = 1, 2, \ldots, M \quad (1)$$

where $s_j(k)$ is the $j$th user’s $k$th symbol. In the presence of a linear dispersive channel (frequency-selective fading or multi-paths) where the receiver collects one sample per chip, the received discrete-time (sampled) signal $\hat{x}_j(n)$ due to user $j$ is

$$\hat{x}_j(n) = \sum_{l=-\infty}^{\infty} x_j(l)g_j(n - d_j - l) \quad (2)$$

where $g_j(n)$ is the effective channel impulse response (IR) sampled at the chip interval $T_c$ (assuming zero transmission delay), and $0 \leq d_j < N$ is the effective transmission delay (mod $N$) of user $j$ in chip periods. The channel IR $g_j(n)$ is assumed to include the effects of chip matched filtering at the receiver. For instance, one may obtain discrete-time $\hat{x}_j(n)$ from continuous-time signal as follows:

$$\hat{x}_j(n) = \int_{nT_c}^{(n+1)T_c} \hat{X}_j(\tau)\psi^*(\tau - nT_c)\,d\tau \quad (3)$$

where the symbol $*$ denotes the complex conjugation operation, $t$ is continuous time

$$\hat{X}_j(t) := \sum_{l} \alpha_{j,l}X_j(t - \tau_{j,l}) \quad (4)$$

$$X_j(t) := \sum_{k=-\infty}^{\infty} s_j(k)\hat{c}_j(t - kT) \quad (5)$$

$$\hat{c}_j(t) := \sum_{i=0}^{N-1} c_j(i)\psi(t - iT_c) \quad (6)$$

$T = NT_c$ is the symbol interval, and $\psi(t)$ is the chip waveform (assumed to be rectangular of duration $T_c$ for simulations.
presented in Section IV). Equation (4) represents the propagation of jth user’s continuous-time baseband transmitted signal X_(j)(t) through a multipath channel with c_(j,l) and τ_(j,l) denoting the complex path gain and total delay (transmission delay plus excess delay), respectively, of the lth path. Equation (3) represents the chip-matched filtering where we do not necessarily have the receiver clock synchronized with the transmitter clock. If the path l = 1 corresponds to the first arrival, then τ_(j,1) is the jth user’s transmission delay with 0 ≤ τ_(j,1) < T, for every j. Although τ_(j,1) is not necessarily an integer multiple of T, the effective chip-rate sampled transmission delay d_(j) in (2) is an integer. (Similarly, τ_(j,l) for any l, is not necessarily an integer multiple of T.)

From (1) and (2), we have

\[ \tilde{x}_j(n) = \sum_{l=-\infty}^{\infty} s_j(l)h_j(n - d_j - lN) \]  

where h_j(n) represents the effective signature sequence of user j (i.e., code c_j(n) “distorted” due to multipath, etc.). The total received signal at chip rate is the superposition of contributions of all users observed in additive white Gaussian noise w(n) as

\[ \tilde{x}(n) = \sum_{j=1}^{M} \tilde{x}_j(n) + w(n). \]  

Finally, collect N measurements of \( \tilde{x}(n) \) into N-vector \( \mathbf{g}(k) = [\tilde{x}(kN), \ldots, \tilde{x}(kN + N - 1)]^T \) to obtain, at the symbol rate, the MIMO model \( \mathbf{u}(k) \) is defined in a manner similar to \( \mathbf{g}(k) \)

\[ \mathbf{g}(k) = \sum_{j=1}^{M} L_j \mathbf{h}_j(k - l) + \mathbf{w}(k) \]  

where \( L_j + 1 \) is the length of the jth user’s vector IR in symbols. \( L_j \) depends upon the multipath delay spread and the transmission delay \( d_j \). In asynchronous CDMA systems, \( d_j \) (transmission delay mod N) is unknown; recall that 0 ≤ \( d_j \) ≤ N - 1 (after sampling). In (10)

\[ \mathbf{h}_j(l) = [h_j(lN - d_j), \ldots, h_j(lN + N - 1)]^T. \]  

Clearly, if \( d_j > 0 \), then \( \mathbf{h}_j(0) \) has its first \( d_j \) components as zero since \( h_j(l) \) [see (8)] is causal.

Let \( d = \max_{1 \leq j \leq M} L_j \). Assume that \( g_j(l) = 0 \) for \( l > mN \) (in addition to \( g_j(l) = 0 \) for \( l < 0 \)) where \( m \geq 1 \) is an integer, i.e., the (excess) multipath delays can be of maximum \( m \) symbol periods (\( mN \) chips). Therefore, \( d = m + 1 \). Using (7), (8), and (11), it then follows that for any \( d_j \geq 0 \)

\[ \mathbf{h}_j^{(d)} := [h_j^{H}(0) \ h_j^{H}(1) \ \cdots \ h_j^{H}(d)]^H = \mathbf{C}_j^{(d)} g_j \]  

where the superscript \( H \) denotes the complex conjugate transpose (Hermitian) operation

\[
\mathbf{C}_j^{(d)} := \begin{bmatrix}
c_j(0) & 0 & \cdots & 0 \\
c_j(1) & c_j(0) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & c_j(N-1) & \cdots & c_j(0) \\
0 & 0 & \cdots & \cdots \\
0 & 0 & \cdots & 0 \\
\end{bmatrix}
\]  

(13)

\[
\mathbf{g}_j := [g_j(-d_j) \ g_j(-d_j + 1) \\
\cdots \ g_j(m + 1)N - d_j - 1]^T
\]  

(14)

\( \mathbf{H}_j^{(d)} \) is \((d + 1)N\)-vector. \( \mathbf{C}_j^{(d)} \) is \([d + 1]N \times [m + 1]N \), and \( \mathbf{g}_j \) is \((m + 1)N\)-vector. Note that most other papers (e.g., [1], [2], and [8]) assume the multipath delays to be only a fraction of the symbol duration, which is not true for high-speed (future) CDMA systems [11], [18]. Note also that not all elements in \( \mathbf{g}_j \) are nonzero. Given the above formulation (i.e., \( 0 \leq d_j < N \) and \( g_j(l) = 0 \) for \( l > mN \) and \( l < 0 \)), it follows that \( \mathbf{h}_j(l) = 0 \) for \( l \geq (m + 2) \).

III. MULTISTEP LINEAR PREDICTORS (MSLP)-BASED SOLUTION

In Section III-A, we discuss existence and calculation of multistep linear predictors for MIMO systems/channels based upon the results of [3] and [4]. In Section III-B, based on [3] and [4], we provide further details on how to use that representation for blind equalization and identification for MIMO channels. In Section III-C, we show how to exploit the known spreading code of the desired user and the results of Sections III-A and III-B to equalize and detect the desired user’s information sequence. The solution of Section III-C is based upon modeling the unknown transmission delay \( d_j \) for user 1 (desired user) as a part of the channel impulse response. In Section III-D, we first consider the case of known delay \( d_j \), and then we propose a solution based upon estimating the unknown \( d_j \). The results of Sections III-A and III-B are from [3] and [4], whereas the material in Sections III-C and III-D is new.

A. Multistep Linear Predictors [3], [4]

Define the \( N \times M \) matrix

\[
\mathbf{H}(l) := \begin{bmatrix}
h_1(l) & \cdots & h_M(l) \\
\end{bmatrix}.
\]  

(15)

Then \( \{\mathbf{H}(l)\}_{l=0}^{L-1} \), \( \mathcal{L} = \max_j \{L_j\} + 1 \) \((=d + 1)\) is the MIMO IR of (6). Recall that in our formulation, \( \mathcal{L} = m + 2 \). Let

\[
\mathcal{H}(z) := \sum_{l=0}^{L-1} \mathbf{H}(l) z^{-l}.
\]  

(16)
Consider the following assumptions on (10) and (16):

\( \text{(A1)} \) \( N > M \).

\( \text{(A2)} \) \( \text{Rank} \{ \mathcal{H}(z) \} = M \forall z \) including \( z = \infty \) but excluding \( z = 0 \), i.e., \( \mathcal{H}(z) \) is irreducible.

\( \text{(A3)} \) The information sequences \( \{ s_j(k) \}_{j=1}^M \) are zero-mean, mutually independent, and temporally white. Take \( E\{ s_j(k)^2 \} = 1 \) by absorbing any nonidentity correlation of \( s_j(k) \) into \( \mathcal{H}(z) \).

\( \text{(A4)} \) \( \{ w(k) \} \) is zero-mean with \( E\{ w(k + \tau)w^H(\kappa) \} = \sigma_w^2 I_{N} \delta(\tau) \), where \( I_{N} \) is the \( N \times N \) identity matrix.

Condition (A1) implies that the number of users are less than the processing gain (number of chips per symbol). Condition (A2) is a technical condition which is milder than that used in [2] and [5] and others. For instance, the full column rank condition of the generalized Sylvester matrix \( \mathcal{H}_{m} \) in [5] implies irreducibility of the corresponding MIMO transfer function \( \mathcal{H}(z) \) [4], [12], [15] but the converse result is not necessarily true [12], [15]. It can be further relaxed as in [3] and [4] by allowing common zeros; we do not pursue this aspect here. Condition (A3) is satisfied by the DS-CDMA information sequences. Condition (A4) is a standard assumption regarding noise at the receiver.

Define the noise-free observations

\[ y_s(k) := \mathcal{H}(z) s(k) = \mathcal{H}(z) \begin{bmatrix} s_1(k) & \cdots & s_M(k) \end{bmatrix}^T. \]  

Let us try to predict the noise-free observations \( I \)-step ahead using linear predictors and past noise-free measurements. Let \( \hat{y}_s(k|k-I) \) denote the linear MMSE estimate of \( y_s(k) \) given \( \{ y_s(n), n \leq k-I \} \). Then \( e(k|k-I) = y_s(k) - \hat{y}_s(k|k-I) \) is the \( I \)-step ahead linear prediction error. It has been shown in [3] and [4] that under (A1)-(A3), finite length predictors exist such that

\[ e(k|k-I) = \begin{bmatrix} n \end{bmatrix}^{T} A^{(I)} \hat{y}_s(k-I) \]

for some \( I \leq M(L-1) + I - 1 \)

\[ (18) \]

where the \( N \times N \) matrices \( A^{(I)} \)'s are the solution to the equation

\[ [ A^{(I)}_1 A^{(I)}_2 \cdots A^{(I)}_N ] = [ R_{ss}(l) \ R_{ss}(l+1) \cdots R_{ss}(P_I) ]^{\#} \]

\[ (19) \]

and \( R_{ss}(l) \), \( R_{ss}(l+1) \), \( \cdots \), \( R_{ss}(P_I) \) denote an \( [NP] \times [NP] \) matrix with its \( i \)-th block element as \( R_{ss}(j-i) = E\{ y_s(k+j-i) y_s^H(k) \} \). Define similarly \( R_{yy}(P_I+1) \), the data-based consistent estimate of \( R_{yy}(P_I+1) \). Therefore, one can estimate the correlation function of the noise-free data \( y_s(k) \) from that of the noisy data. Furthermore, one can obtain consistent estimates of \( A^{(I)} \)'s using the correlation function of \( y_s(k) \) and (20).

The representation (23) can be used to perform various desirable tasks, such as blind channel estimation, equalization, MUI suppression, etc. Note that since the exact value of \( P_I \) is typically unknown, one may overfit and take \( P_I \geq M(L-1) + I - 1 \), in which case some \( A^{(I)} \)'s will be zero. This, however, does not affect (21) and (22).

**B. Blind Equalization/Identification for MIMO Channels**

We wish to design an MMSE (minimum mean-square error) linear equalizer of a specified length to extract the desired user’s signal, say \( s_1(k) \), and to simultaneously suppress the MUI (contributions due to \( s_j(k), j = 2, 3, \ldots, M \)). It is not too hard to establish (using the orthogonality principle, for example) that the \( 1 \times N \) MMSE vector equalizer \( [ f^{H}(l) ]_{l=0}^{L-1} \) of length \( L_e \) satisfies

\[ f^{H}(l) = \begin{bmatrix} h^{H}(0) & h^{H}(1) & \cdots & h^{H}(L_e-1) \end{bmatrix} \]

\[ (24) \]

assuming that \( d \leq L_e - 1 \) where, recall that, \( R_{yy}L_e \) is an \( [NL_e] \times [NL_e] \) matrix with its \( i \)-th block-element given by

where the superscript \( \# \) denotes the pseudoinverse. Furthermore, the prediction error \( e(k|k-I) \) is given by

\[ e(k|k-I) = \sum_{i=0}^{l} H(i) s(k-i) \]

\[ (21) \]

such that

\[ \mathbb{E} \{ e(k|k-I) y_s^H(k-I) \} = 0 \] \( \forall n \leq l \).

(22)

Thus, by [3] and [4], under (A1)-(A3), \( y_s(k) = \mathcal{H}(z) s(k) \) has the (canonical) representation \( (l = 1, 2, \ldots) \)

\[ y_s(k) = \sum_{i=1}^{n} A^{(l)}_{k} y_s(k-i) + e(k|k-l) \]

for some \( P_I \leq M(L-1) + I - 1 \).

(23)
The equalized output is given by

\[ \hat{s}_1(k - \overline{d}) = \sum_{i=0}^{L-1} J^H(i) y(k - i). \]  

(25)

Clearly one can obtain a consistent estimate of \( R_{yyU} \) from the given data. It remains to estimate \( h_1(l) \)'s to complete the design. This is where the multistep predictor approach turns out to be useful; we discuss this in the next part of the paper, we use \( \overline{d} = d = \max_{1 \leq j \leq M} L_j \). This choice allows exploitation of the entire effective code signature sequence of the desired user in (24).

As noted earlier, given noisy measurements \( y(k) \), one can consistently estimate noise variance \( \sigma_n^2 \) under (A4), using the correlation function of \( y(k) \). Therefore, one can estimate the correlation function of the noise-free data \( y_s(k) \) from that of the noisy data. In the following discussion it is assumed that such is the case. Furthermore, one can obtain consistent estimates of \( \tilde{A}_k^{(l)} \)'s using the correlation function of \( y_s(k) \) and (20). Define

\[ \tilde{L} = K + d + 1 \quad \text{where} \quad K := P_l - l, \]  

(26)

By (23) and (26), \( K + 1 \) is the predictor length for any \( l \geq 1 \), independent of \( l \), such that \( K \leq M(\overline{L} - 1) - 1 \). Rewrite (23) as

\[ e(k|k - l) = \sum_{i=0}^{l} \tilde{A}_k^{(i)} y_s(k - i) \]  

(27)

where \( l \geq 1 \)

\[ \tilde{A}_k^{(l)} = \begin{cases} I_N, & \text{for } i = 0 \\ 0, & \text{for } 1 \leq i \leq l - 1 \\ -A_k^{(l)}, & \text{for } l \leq i \leq K + l \\ 0, & \text{for } K + l + 1 \leq i \leq \tilde{L}. \end{cases} \]  

(28)

By (26), \( P_l = K + l \); therefore, for each \( l \), we estimate \( K + 1 \) coefficients in (20). For \( l \geq 2 \), define

\[ e_l(k) := e(k|k - l) - e(k|k - l + 1) = \sum_{i=0}^{l} D_i^{(l-1)} y_s(k - i) \]  

(29)

where \( l \geq 2 \)

\[ D_i^{(l-1)} := \tilde{A}_k^{(i)} - \tilde{A}_k^{(l-1)}, \quad i = 0, 1, \ldots, \tilde{L}. \]  

(30)

By (28), \( D_0^{(l-1)} = 0 \quad \forall l \geq 2 \)

Consider the \([N(d + 1)\)-vector

\[ E(k) := \begin{bmatrix} e_{d+1}^T(k + d) \\ e_{d}^T(k + d - 1) \\ \vdots \\ e_{d+1}^T(k + 1) \\ e^T(k|k - 1) \end{bmatrix}^T. \]  

(31)

Using (21) and (27)–(31), we have

\[ E(k) = \mathcal{D} Y_s(k) = \begin{bmatrix} H(d) \\ H(d-1) \\ \vdots \\ H(0) \end{bmatrix} s(k) = \tilde{H} s(k) \]  

(32)

where the \([N(K + 2d + 1)\]-vector \( Y_s(k) \) is given by

\[ Y_s(k) := \begin{bmatrix} y_{d+1}^T(k + d - 1) \\ \vdots \\ y_s^T(k - K - d - 1) \end{bmatrix}^T \]  

(33)

and (34), shown at the bottom of the page, is an \([N(d + 1) \times \frac{N(\overline{L} + d)}{2}] \) matrix. In (34), we have used the fact that \( D_0^{(\overline{L}-1)} = 0 \quad \forall l \geq 2 \). The representation (32) for \( s(k) \) is fundamental.

Using (A3), it follows that:

\[ R_{EE}(0) = E \{ E(k) E^H(k) \} = \tilde{H} \tilde{H}^H = \mathcal{D} R_{ss} \mathcal{D}^H. \]  

(35)

Clearly, rank \( \tilde{H} \tilde{H}^H = M \) as rank \( \mathcal{H}(0) = M \) by (A2). Since \( \tilde{H} \tilde{H}^H = \mathcal{H} \mathcal{H}^H \) for any unitary \( \mathcal{U} \) (i.e., \( \mathcal{U} \mathcal{H}^H = \mathcal{I}_M \)), one cannot uniquely determine \( \tilde{H} \) from (35) given the data correlation function and \( \mathcal{D} \); one needs to exploit the higher order statistics of the data [3], [4]. This is where the knowledge of the desired user’s spreading code turns out to be useful. Calculate \( R_{EE}(0) \) as

\[ R_{EE}(0) = \mathcal{D} \left[ R_{yy(K + 2d + 1)} - \sigma_n^2 I_{N(K + 2d + 1)} \right] \mathcal{D}^H. \]  

(36)
C. Code-Constrained Solution: Unknown Delay

Consider (32). By the discussion following (35), an eigenvalue decomposition (EVD) of the correlation matrix in (36) defines a “signal” subspace of dimension $M$ and a “noise” subspace of dimension $N_d + N - M$. That is,

$$
\mathcal{D} \left[ R_{yy}(K+2d+1) - \sigma_0^2 I_{N(N+2d+1)} \right] \mathcal{D}^H
= [\mathcal{U}_s \ \mathcal{U}_n] \begin{bmatrix}
\Lambda_s & 0 \\
0 & \Lambda_n
\end{bmatrix} [\mathcal{U}_s \ \mathcal{U}_n]^H
$$

(37)

where $\Lambda_s = \text{diag}(\lambda_1, \ldots, \lambda_M)$ contains the $M$ largest eigenvalues in descending order, $\mathcal{U}_s = [\mathbf{u}_1 \cdots \mathbf{u}_M]$ contains the $M$ corresponding orthonormal eigenvectors defining the signal subspace, $\Lambda_n = \alpha I_{N(N+2d+1)} + \sigma_0^2 I_M (\alpha \to 0)$, and $\mathcal{U}_n = [\mathbf{u}_{M+1} \cdots \mathbf{u}_{N_d+N}]$ contains the $N_d + N - M$ orthonormal eigenvectors corresponding to the noise subspace (ideally eigenvalues equaling zero). Therefore, we should be able to exploit subspace methods based on the EVD of (37), instead of the EVD of the data correlation matrix as in [5], [6], [10], [19], etc., or instead of the SVD (singular value decomposition) of a large data matrix as in [2].

It follows from (35)–(37) that $\mathbf{H}$ is orthogonal to $\mathbf{sp} \{\mathcal{U}_n\}$, the subspace spanned by the columns of $\mathcal{U}_n$. Define an $[N(d+1)] \times [N(d+1)]$ matrix $T_d$ as

$$
T_d :=
\begin{bmatrix}
0 & \cdots & 0 & I_N \\
0 & \cdots & I_N & 0 \\
0 & \cdots & 0 & I_N \\
\vdots & \cdots & \vdots & \vdots \\
I_N & \cdots & 0 & 0
\end{bmatrix}.
$$

(38)

Further define the $[N(d+1)]$-column vector

$$
\hat{\mathbf{h}}_j^{(d)} := \left[ \mathbf{h}_j^H(d) \quad \mathbf{h}_j^H(d-1) \quad \cdots \quad \mathbf{h}_j^H(0) \right]^H.
$$

(39)

It then follows that $\hat{\mathbf{h}}_j^{(d)}$ is the $j$th column of $\hat{\mathbf{H}}$. Moreover, by (12), (38), and (39), we have $\hat{\mathbf{H}}_j^{(d)} = T_d \mathbf{h}_j^{(d)}$. By orthogonality of $\hat{\mathbf{H}}$ to $\mathbf{sp} \{\mathcal{U}_n\}$, it follows that

$$
\hat{\mathbf{h}}_j^{(d)H} \mathbf{u}_l = \mathbf{h}_j^{(d)H} T_d^H \mathbf{u}_l = 0, \quad l = M+1, M+2, \ldots, N_d+N.
$$

(40)

Finally, using (12) and (40), we have

$$
\mathbf{g}_l^H \left[ C_j^{(d)H} T_d^H \mathbf{u}_l \right] = 0, \quad l = M+1, M+2, \ldots, N_d+N.
$$

(41)

For the desired user ($j = 1$), $C_1^{(d)}$ is known (and so is $T_d$) and $\mathcal{U}_n$ can be estimated via (37). Therefore, an estimate of the desired user’s multipath channel $\mathbf{g}_1$ can be obtained by minimizing the cost

$$
\mathbf{g}_1^H \left( C_1^{(d)H} T_d \left[ \sum_{l=M+1}^{N_d+N} \mathbf{u}_l \mathbf{u}_l^H \right] T_d C_1^{(d)} \right) \mathbf{g}_1 =: \mathbf{g}_1^H A \mathbf{g}_1
$$

subject to the constraint $\mathbf{g}_1^H \mathbf{g}_1 = 1$. The solution (up to a scale factor) is given by the eigenvector corresponding to the smallest eigenvalue of the matrix $A$. Once $\mathbf{g}_1$ is estimated, we can obtain $h_1^{(d)}$ via (12) with $j = 1$ and then implement the MMSE equalizer (24). The uniqueness (identifiability) of the solution to (42) is addressed in Section III-C1.

A brief outline of the proposed approach is as follows. 

1) Assuming that the carrier frequency is known (i.e., has been acquired), carry out noncoherent carrier demodulation to obtain continuous-time baseband signal. Perform chip-matched filtering as in (3) to obtain chip-rate discrete-time data.

2) Estimate the data correlation matrix $R_{yy}$ by sample averaging of the chip-rate discrete-time data. Perform an EVD to estimate the noise variance $\sigma_0^2$ as the average of the smallest $N_d + N - M$ eigenvalues; this may be taken as the smallest eigenvalue if $M$ is unknown. Estimates of the correlation function of $y_k(k)$ then easily follow. See [4] for more details.

3) Use (20) to estimate the matrix coefficients $A_j^{(d)}$s of the various order predictors.

4) Construct the matrix $\mathcal{D}$ in (34) using the estimates of $A_j^{(d)}$s, (28) and (30).

5) Perform the EVD as in (37). Estimate the desired user’s (user 1) multipath channel $\mathbf{g}_1$, as the eigenvector corresponding to the smallest eigenvalue of the matrix $A$ defined in (42).

6) Estimate $h_1^{(d)}$ up to a scale factor via (12) with $j = 1$ and then implement the MMSE linear equalizer (24).

Note that the proposed solution recovers the desired information sequence only up to a scale factor (phase ambiguity). In practice, this problem can be alleviated via differential encoding at the transmitter and differential decoding at the receiver [17].

1) Identifiability: Now we investigate the conditions [in addition to (A1)–(A4)] under which the solution of Section III-C will yield the desired solution. Consider

(A5) The $[N(d+1)] \times [(m+1)N + M - 1]$ matrix

$$
[ C_1^{(d)} : T_d h_2 : T_d h_3 : \cdots : T_d h_M ]
$$

has full column rank.

Claim: Suppose that $2N$-vector $\mathbf{g}$ minimizes $\mathbf{g}^H A \mathbf{g}$ [see (42)] subject to $\mathbf{g}^H \mathbf{g} = 1$. Then, under (A1)–(A5), $\mathbf{g} = \alpha \mathbf{g}_1$ for some $\alpha \neq 0$ where $\mathbf{g}_1$ satisfies (12) for $j = 1$.

Proof: By construction, $\mathbf{g}$ satisfies

$$
\mathbf{u}_l^H T_d C_1^{(d)} \mathbf{g} = 0, \quad l = M+1, M+2, \ldots, N_d+N.
$$

Therefore, it follows that:

$$
T_d C_1^{(d)} \mathbf{g} \in \mathbf{sp} \{ \mathbf{H} \} \Rightarrow C_1^{(d)} \mathbf{g} \in \mathbf{sp} \{ \mathbf{h}_1^{(d)}, \ldots, \mathbf{h}_M^{(d)} \}.
$$

(44)

Under (A5), (44) is possible only for $C_1^{(d)} \mathbf{g} = \alpha h_1^{(d)}$ for some $\alpha \neq 0$. The desired result then follows by invoking (12) for $j = 1$. □

Remark: Condition (A5) is the same as the condition (C5) in [5, Proposition 2] after accounting for the asynchronous case and some notational differences. It is also equivalent to that stated in [2, Theorem 1] after accounting for some notational differences.
D. Code-Constrained Solution: Estimated Delay

The solution of Section III-C is based upon modeling the unknown transmission delay \(d_1\) for user 1 (desired user) as a part of the channel impulse response \(g_1\). One can use the cost (42) to estimate the delay \(d_1\) (in a manner similar to that in [2] and [5]), thereby reducing the number of unknowns in \(g_1\) from \((m+1)N\) to \(mN + 1\). The anticipated benefit is improved accuracy, as fewer parameters are estimated.

Suppose that \(d_1\) were known. Then the first \(d_1\) and the last \(N - d_1 - 1\) elements of \(g_1\) [see (14)] are known to be zeros. In this case, we need to estimate a smaller size \((mN + 1)\) elements multipath channel. Let \(g_1\) denote an \(mN + 1\)-column vector obtained from \(g_1\) by deleting the known null entries, and let \(C_1^{(d_1)}\) denote a \([N+1] \times [mN + 1]\) submatrix of \(C_1^{(d)}\) obtained by deleting the first \(d_1\) and the last \(N - d_1 - 1\) columns of \(C_1^{(d)}\). Then \(C_1^{(d)} g_1 = C_1^{(d_1)} g_1\) under the aforementioned scenario. Following (42), the multipath channel \(g_1\) may be estimated by minimizing the cost

\[
g_1^H \left( C_1^{(d_1)} C_1^{(d_1)H} \right)^{-1} \hat{g}_1 =: \hat{g}_1, \tag{45}
\]

subject to the constraint \(\hat{g}_1^H \hat{g}_1 = 1\). The rest of the details are as in Section III-C.

In practice (and also under the assumptions of this paper), the transmission delay \(d_1\) may not be (is not, respectively) known. Following [2] and [5], we may estimate the delay \(d_1\) as \(\hat{d}_1\) given below:

\[
\hat{d}_1 := \arg \min_{0 \leq d_1 \leq N-1} \min_{\|g_1\|_2 = 1} \left\| \hat{g}_1 \right\|_2^2, \tag{46}
\]

The rationale for the above procedure is as in [2] and [5]: under the correct choice of \(d_1\), the cost function should be a minimum. Finally, \(h_1^{(d)} = C_1^{(d)} g_1 = C_1^{(d_1)} g_1\), which is then used in (24) to implement the MMSE equalizer for user 1.

IV. SIMULATION EXAMPLES

In this section, we consider two simulation examples to illustrate the proposed approach and to compare it with the approaches of [2], [5], [6], and [19]. Note that the approaches of [2] and [5] are essentially the same. In [2], one has the choice of extracting the signal subspace using a data matrix \(X\), via SVD of \(X\) or EVD of \(XX^H\). In [5], the same signal subspace is extracted via EVD of a data correlation matrix which is the same as \(XX^H\). In our simulations, we have used the approach of [5] in exploiting the data correlation matrix. In extracting the signal subspace, one has to determine the effective rank of this data correlation matrix, say \(R\). In [2] and [5], the rank of this matrix has been specified in terms of several unknown parameters: number of active users, channel length for the various users, etc., provided that the generalized Sylvester matrix for the underlying MIMO channel is of full column rank. Use of this theoretical rank did not work for the simulation examples considered in this section. In this paper, we determine the effective rank as number of effectively nonzero singular values (or eigenvalues) of the relevant matrix. Suppose that a correlation matrix \(R\) is \(Q \times Q\). Let \(\lambda_i \geq 0 (i = 1, 2, \ldots, Q)\) denote its eigenvalues (or singular values) in descending order of magnitude. The rank \(\tilde{n}\) of \(R\) is determined as the smallest \(n\) for which

\[
\tilde{n} := \left\lceil \frac{\sum_{i=n+1}^{Q} \lambda_i}{\sum_{i=1}^{Q} \lambda_i} \leq \epsilon \right\rceil \tag{47}
\]

where \(\epsilon > 0\) is a small number (threshold). The same criterion was used to calculate the effective rank and the pseudoinverse in (20). Moreover, \(R_{\text{mmse}}^{-1}\) in (24) was also computed using pseudoinverse via EVD with with rank determination using (47) (with \(\epsilon = 0.01\)). Thus, calculation of \(R_{\text{mmse}}^{-1}\) was regularized (see also [4] and [5]).

The normalized equalization mean-square error (NEMSE) (normalized by the desired user’s information sequence power) and the probability of symbol detection error (\(P_e\)) after equalization were taken as the two performance measures after averaging over 100 Monte Carlo runs. Since lower MSE does not necessarily imply lower \(P_e\) (because the former measures an average quantity, whereas the latter is strongly influenced by the probability distribution of the noise and residual intersymbol interference at the equalizer output), and since use of MSE as a performance measure is widespread (see [1], [6], [8], and [19], for instance), we use both of these performance measures to illustrate our simulation results. The equalized data were rotated and scaled before calculating the two performance measures. After designing the equalizers based on the given data record, the designed equalizer was applied to an independent record of length 3000 symbols in order to calculate normalized MSE and \(P_e\). Therefore, the estimated \(P_e\) is not reliable below approximately \(10^{-4}\), hence, these values are not shown in the figures to follow.

In Examples 1 and 2, for a baseline comparison, we also simulate an ideal (clairvoyant) matched filter receiver which is matched to the true effective signature sequence \(h_1(n - d_1)\) for \(h_1(I)\), see (1) and (8)] of user 1. This matched filter has information (e.g., channel for user 1 including transmission delay) which is not available to other approaches. In practice, one would require some means to estimate this information. For a further baseline comparison, we also simulate a linear MMSE filter receiver (24) with perfectly known channel impulse response for the desired user (called “known channel MMSE filter” hereinafter). The data correlation matrix in (24) is estimated from the data, and its inverse is computed as discussed earlier, in order to implement the known channel MMSE filter.

A. Example 1: 16 Chips/Symbol, Five Users, Max. Excess Delay of OneSymbol

We consider the case of five users, each transmitting four-QAM signals, and short-codes with 16 chips per symbol. The spreading codes were randomly generated binary (±1, with equal probability) sequences. The multipath channels for each user have four paths with transmission delays uniformly distributed over one symbol interval, and the remaining three multipaths having mutually independent delays (with respect to
the first arrival) uniformly distributed over one symbol interval \(m_1 = 1\) in (14). The first arrival was assumed to be line-of-sight (direct) arrival with its amplitude normalized to one, and the remaining three multipath amplitudes were mutually independent, complex Gaussian with zero-mean and standard deviation of 0.3 (such multipath channel has been considered in [9]). The channels for each user were randomly generated and then fixed for all 100 Monte Carlo runs. Complex white-zero mean Gaussian noise was added to the received signal from the three users. The SNR refers to the receiver SNR of the desired user, which was user 1, and it is given by [see (7)–(10) and (A4)]

\[
\text{SNR} := \frac{\sum_{n=1}^{N} E[\{|\tilde{x}_1(n)|^2\}]}{N\sigma_w^2}.
\]

In the equal-power case (zero-dB MUIs), all users have the same power incident at the receiver, i.e., \(\sum_{n=1}^{N} E[\{|\tilde{x}_j(n)|^2\}]\) is the same \(\forall j\). In the near–far case (ten-dB MUIs), the desired user power (i.e., \(\sum_{n=1}^{N} E[\{|\tilde{x}_1(n)|^2\}]\) is 10 dB below that of other users. We should note that the SNR is defined differently in [5]. Translated to our model (7)–(10), the SNR in [5] is defined (see [5, p. 101]) as \(\text{SNR} = E[\{|\tilde{x}_1(n)|^2\}]/\sigma_w^2\), which is the transmitted power of the desired user divided by the noise (two-sided) power spectral density. Using SNR to denote the definition of (48), and \(\text{SNR}_{[5]}\) to denote the definition of [5], for our model (7)–(10), the two are related via

\[
\text{SNR} = \text{SNR}_{[5]} \sum_{n=0}^{\infty} |h_1(n-d_1)|^2.
\]

Since [5] normalizes the amplitude of the first arrival to one \((f(0) = 1)\) in the notation of [5, Example 2], \(h_1(0) = 1\) in the notation of this paper, it follows that \(\text{SNR} > \text{SNR}_{[5]}\) for the same example; i.e., the SNR stated in [5] translates to a (much) higher “true” receiver SNR for the desired user for examples having significant multipaths.

Equalizer of length \((L_e)\) 4 symbols and desired delay \((\lag)\) \(d = d_2 = 2\) was designed using the proposed algorithm (both the unknown delay case, as in Section III-C, and the estimated delay case, as in Section III-D). In designing the proposed approach, we took \(K = 3\) and \(I_1 = K + l\) in (20) and (32). The pseudoinverse in (20) was calculated via EVD (see also [4]) with rank determination using (47) (with \(\epsilon = 0.01\)). In applying the proposed approach, we did not assume knowledge of \(M\), the number of users. It is needed in defining \(U_d\), see (37). We estimated \(M\) by the rank \(\hat{M}\) of \(R_{EE}(0)\) [see (36)] which was taken to be the number of effectively nonzero singular values of \(R_{EE}(0)\) with \(\epsilon\) in (47) set to 0.02, with the restriction that \(\hat{M} \geq N\). The approach of [5] (equivalent to that of [2], as noted earlier) was also simulated with a “smoothing factor” \(m\) in [5] of 4 \((=L_{e})\) with rank determination of the data correlation matrix in [5] using (47) (with \(\epsilon = 0.01\), as for the proposed approach). The approach of [5] was used to estimate the desired user’s channel IR which, in turn, was used in the MMSE equalizer (24). We also applied the approach of [6], [19] using equalizer of length 4 symbols and desired delay \(\lag = d = 2\). To determine the rank of the data covariance matrix \(R\) in [6, (13)], [19] we use EVD and take the rank as the number of effectively nonzero eigenvalues with \(\epsilon\) in (47) set to 0.017. Finally, the approach of [8] did not work \((J_0 > 0.3\) for all cases) for the considered example, perhaps because the multipath spread is comparable to symbol duration in this paper, instead of being a fraction of the symbol duration as in [8].

Figs. 1 and 2 show the results for various SNR’s for the equal power case, and Figs. 3 and 4 show the same for the near–far sce-
The approach of [2] and [5] is sensitive to the (unknown) rank of the correlation matrix. As seen in Figs. 1 and 2, the performance of [2] and [5] deteriorates below the SNR = 20 dB. Note that such a deterioration (due to sensitivity to rank estimation) is not seen in the examples of [2] and [5]. Reference [2] simulates very low delay spreads so that their examples are not comparable to our example. [5] simulates examples similar to ours; however, there are two main differences. As noted earlier in this section [see (48) and (49)], the SNR definition of [5] underestimates the true SNR at the receiver. As [5] does not specify the details of their multipath generation (how many paths, etc.), it is not possible to say what the true SNRs are in [5, Example 2]. Moreover, in [5, Example 2], all multipaths have random amplitudes, whereas in our example, we have a direct arrival.
Fig. 4. Example 1: Probability of symbol error for user 1; rest as for Fig. 3.

with unit amplitude and three later arrivals with random amplitudes. These two differences make it hard to compare our results with the results of [5, Example 2].

Figs. 1–4 also show that the two versions of the proposed approach (unknown transmission delay considered in Section III-C, and estimated delay considered in Section III-D) are not too far apart, with the estimated delay version performing better as fewer channel coefficients are estimated in this case. Moreover, the estimated delay version has a performance quite close to that of the known channel MMSE filter. The performance of the ideal matched filter is quite sensitive to the near–far problem. Overall, it is seen that the proposed approach is significantly better than the approaches of [2], [5], [6], and [19].

**Computational Complexity:** We also compared the computational complexity of the various approaches with regard to the number of floating point operations (FLOPS) needed to execute one simulation run for Example 1 using MATLAB, involving MMSE equalizer design and equalization execution. In the case of the ideal matched filter, there was no filter/equalizer to be designed and, therefore, the FLOP count is for filter execution (implementation) only. Table I shows the FLOP count for SNR = 15 dB and equal power case. It is seen that the computational complexity of the proposed approaches is comparable to that of [2], [5], [6], and [19].

**Example 2: Eight Chips/Symbol, Three Users, Max. Excess Delay of Two Symbols**

We consider the case of three users, each transmitting four-QAM signals, and short-codes with eight chips per symbol. The spreading codes were randomly generated binary (±1, with equal probability) sequences. The multipath channels for each user have four paths with transmission delays uniformly distributed over one symbol interval, and the remaining three multipaths having mutually independent delays (with respect to the first arrival) uniformly distributed over two symbol intervals \(m = 2\) in (14). The other details regarding the channel are as in Example 1. Equalizers of length \(L_e\) five symbols and desired delay (lag) \(d = d = 3\) were designed for the proposed approaches; the approaches of [2], [5], [6], and [19] were not simulated. Thresholds for rank determination, etc., were exactly as for Example 1. Figs. 5 and 6 show the simulation results. It is seen that the proposed approaches work with larger ISI spreads. In this case, the performance of the estimated delay version is inferior to that of the known channel MMSE filter at lower SNRs but close to it for high SNRs.

**Example 3: 12 Chips/Symbol, Variable Number of Active Users**

In this example, we fix the (desired user’s) SNR at 15 dB and vary the number of active users with processing gain equal to 12. The rest of the details regarding the channel and codes are as for Example 1: random binary spreading codes with 12 chips/symbol, channel with (line-of-sight) direct arrival with amplitude normalized to one, and the remaining three multipaths having complex Gaussian amplitudes (mean zero, standard deviation 0.3), transmission delays uniformly distributed over one symbol duration, and remaining multipath delays (relative to the
Equalizers of length ($L_e$) four symbols and desired delay (lag) $d = 2$ were designed for all approaches. The smoothing factor for the approach of [5] was set to $L_e = 4$. Thresholds for rank determination, etc., were exactly as for Example 1. None of the approaches had the knowledge of the number of active users.

The simulation results are shown in Figs. 5 and 6. It is seen that, overall, the proposed approaches (both versions) outperform [2], [5], [6], and [19]. The observations made in Example 1 regarding the various approaches apply here too. It is also seen that the near–far resistance starts to break down as the number of active users increases.
V. CONCLUSION

A multistep linear prediction approach was presented for blind channel estimation, MUI suppression, and detection of asynchronous short-code DS-CDMA signals in multipath channels. Only the spreading code of the desired user was assumed to be known. Its transmission delay was unknown. We exploited the recently proposed MSLP approach [3], [4] for blind MIMO channel estimation in conjunction with the structure imposed by the desired user’s spreading code sequence. With the knowledge of the desired user’s code sequence, only the second-order statistics of the data were needed under certain sufficient conditions on the underlying multiuser MIMO transfer function.

Three illustrative simulation examples were presented where the proposed approaches (two versions) were compared to the approaches of [2], [5], [6], and [19]. For the presented examples, the proposed approaches outperform the other approaches. The approach of [8] was found not to work for the considered examples.

REFERENCES


Jitendra K. Tugnait (M’79–SM’93–F’94) was born in Jabalpur, India, on December 3, 1950. He received the B.Sc. (Hons.) degree in electronics and electrical communication engineering from the Punjab Engineering College, Chandigarh, India, in 1971; and the M.S. and E.E. degrees from Syracuse University, Syracuse, NY; and the Ph.D. degree from the University of Illinois, Urbana-Champaign, in 1973, 1974, and 1978, respectively, all in electrical engineering.

From 1978 to 1982, he was an Assistant Professor of Electrical and Computer Engineering at the University of Iowa, Iowa City. He has also been associated with the School of Radar Studies, Indian Institute of Technology, New Delhi, India, and Space Applications Centre, Ahmedabad, India, during 1975–1976. He was with the Long Range Research Division of the Exxon Production Research Company, Houston, TX, from June 1982 to September 1989. He joined the Department of Electrical and Computer Engineering, Auburn University, Auburn, AL, in September 1989 as a Professor. His current research interests are in statistical signal processing, digital communication and stochastic systems analysis.

Dr. Tugnait served as an Associate Editor of the IEEE TRANSACTIONS ON AUTOMATIC CONTROL during 1985 and 1986, as an appointed member of the Board of Governors of the IEEE Control Systems Society in 1986, as a member of the Operating Committee of the 1991 CDC in Brighton, U.K., and as an Associate Editor of the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 1994 to 1997. He is currently a member of the signal processing for communications technical committee of the IEEE Signal Processing Society. In 1996 the Institution of Electrical Engineers awarded him the IERE Benefactors Premium for his paper in the December 1994 issue of Proceedings of the Institution of Electrical Engineers-Communications. He is the recipient of the 1997 Auburn Alumni/Sigma Xi Outstanding Faculty Research Award, the 2000–2005 Auburn Alumni Professorship from Auburn University, and the 2000 Senior Faculty Research Award from the College of Engineering of Auburn University.

Tong-tong Li (S’97) was born in Xi’an, China. She received the M.S. and Ph.D. degrees from the Department of Electrical and Computer Engineering, Auburn University, Auburn, AL, in 1998 and 2000, respectively.

Since September 2000, she has been with Bell-Labs, Lucent Technologies, Inc., Holmdel, NJ. Her research interests include digital communications, digital signal processing, signal detection, and blind equalization.