

ENERGY EFFICIENT MULTI-HOP WIRELESS BACKHAUL IN HETEROGENEOUS CELLULAR NETWORKS

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ABSTRACT

This paper considers multi-hop wireless backhaul in randomly distributed heterogeneous cellular networks. Following the framework of stochastic geometry, we model the distribution of access points with wireless backhauls and that of base stations with dedicated wired backhauls as two independent Poisson Point Processes. A routing protocol that aims to maximize the network throughput is proposed. To improve the spectral efficiency, intra-cell resource reuse is applied. A tractable interference model is established by taking both the inter-cell and intra-cell interferences into consideration. Our performance analysis shows that the proposed scheme is a more energy efficient alternative to the single-hop backhaul.

Index Terms— Heterogeneous cellular network, wireless backhaul, multi-hop relay, stochastic geometry

1. INTRODUCTION

The deployment of small low power base stations transforms the macrocell-based cellular network into a heterogeneous one. The densification of small cells necessitates the densification of backhaul connections; however, due to geographic limit, it may not be cost-effective, or even possible, to provide each small base station with a wired backhaul. One possible solution is wireless backhaul, where small base stations backhaul their data through the wireless links to their neighbors, which have wired connections to the core network [1].

In this paper, we use *access point* (AP) to refer to a small base station with only wireless backhaul and *base station* (BS) to refer to one equipped with dedicated wired backhaul. We use *cell* to denote the area served by a BS and/or its associating APs.

We propose to explore the performance analysis of cellular networks with multi-hop wireless backhaul using stochastic geometry [9]. As a powerful mathematical tool in network performance analysis and optimization, stochastic geometry has been widely applied in the modeling of heterogeneous networks in recent years [2] [3] [4]. In [5] [6], stochastic geometry was used to analyze relay-aided *two-hop cellular networks*. In [7] [8], it was used to analyze multi-hop transmission in *ad-hoc networks*. In this paper, we extend the analysis to cellular networks with multi-hop wireless backhaul by identifying two major challenges: routing selection and interference modeling. *First*, we propose and analyze a distributed routing protocol. *Then*, we utilize stochastic geometry methods to model the interference that APs are subject to. Combining the results of these two parts, we calculate the throughput and spectral efficiency of multi-hop backhaul. Numerical results show that the proposed

scheme achieves a notable improvement on spectral efficiency over the single-hop wireless backhaul under the scenarios where the BSs are sparsely deployed or the transmit power is limited.

2. SYSTEM DESCRIPTION

2.1. Network Description

We consider a heterogeneous cellular network consisting of BSs, APs and end-point users. Since we primarily focus on wireless backhaul, it is assumed that the communication between users and BSs/APs are over a different spectrum from the one used by wireless backhaul. As a result, the interference from users to wireless backhaul can be avoided.

Let $\Phi_{BS} = \{\mathbf{X}_{BS,i}\}$ and $\Phi_{AP} = \{\mathbf{X}_{AP,j}\}$ denote the sets of BSs and APs, respectively, where $\mathbf{X}_{BS/AP,i/j}$ is the random location of the i/j th BS/AP. Φ_{BS} and Φ_{AP} are modeled as two independent homogeneous Poisson Point Processes (PPPs) with intensities λ_{BS} and λ_{AP} , respectively. Each BS/AP is equipped with two omni-directional antennas, one for transmission and the other for reception, and transmits signals with power P_T . It is assumed that AP $\mathbf{X}_{AP,j}$ establishes a multi-hop wireless backhaul to BS $\mathbf{X}_{BS,i}$ via the relay of other APs. Let $\mathbf{X}_{AP,R_{j,i}(n)}$ denote the n th relay from $\mathbf{X}_{AP,j}$ to $\mathbf{X}_{BS,i}$ and $N_{j,i}$ the number of hops needed. In particular, $\mathbf{X}_{AP,R_{j,i}(0)} = \mathbf{X}_{AP,j}$ and $\mathbf{X}_{AP,R_{j,i}(N_{j,i})} = \mathbf{X}_{BS,i}$. In the routing process that we will propose, the selection of the next relay $\mathbf{X}_{AP,R_{j,i}(n+1)}$ is determined by $\mathbf{X}_{AP,R_{j,i}(n)}$. Each AP will search for a route to its nearest BS, and we denote the set of all the APs that backhaul to BS $\mathbf{X}_{BS,i}$ by $\Phi_{AP}^{\mathbf{X}_{BS,i}}$.

The total available spectrum is divided into resource blocks (RBs). For simplicity, all the RBs are assumed to have the same bandwidth. Throughout the paper, a saturated traffic load model is assumed, where all the APs are active and demand the same amount of bandwidth for backhaul. The multi-hop wireless backhaul operates in a virtual circuit mode. For $0 \leq n \leq N_{j,i} - 1$, each node $\mathbf{X}_{AP,R_{j,i}(n)}$ will take two RBs to set up the communication links with the next relay node $\mathbf{X}_{AP,R_{j,i}(n+1)}$, one for the uplink $\mathbf{X}_{AP,R_{j,i}(n)} \rightarrow \mathbf{X}_{AP,R_{j,i}(n+1)}$ using $RB_{j,i}^u(n)$, and the other for the downlink $\mathbf{X}_{AP,R_{j,i}(n+1)} \rightarrow \mathbf{X}_{AP,R_{j,i}(n)}$ using $RB_{j,i}^d(n)$. Each virtual link may be subject to the interference generated by other nodes (either BS or AP) transmitting over the same RB.

To improve the spectral efficiency, RBs can be partially reused within one cell. In general, the APs that are close to the BS are likely to have more virtual links than those farther away, because inner APs have to relay the packets of the outer ones. To reduce intra-cell interference due to RB reuse, available RBs in the cell of $\mathbf{X}_{BS,i}$ are divided into two sets $S_{RB,i}^1$ and $S_{RB,i}^2$, where the RBs in $S_{RB,i}^1$ are allocated to the APs within distance r_b from the BS and

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cannot be reused in the same cell, while RBs in $S_{RB,i}^2$ are used by virtual links out of that region and can be reused. Based on the ratio of the number of RBs to the number of virtual links required, if on average, each RB in $S_{RB,i}^2$ is reused μ times, then μ is called the resource reuse factor of the system. To minimize the interference from/to the BS, the BS only uses RBs in $S_{RB,i}^1$.

For any $\mathbf{X}_{AP,j} \in \Phi_{AP}^{\mathbf{X}_{BS,i}}$, consider the backhaul from $\mathbf{X}_{AP,j}$ to $\mathbf{X}_{BS,i}$. Let $N_{j,i}^1$ denote the number of hops within $\mathcal{B}_{\mathbf{X}_{BS,i}}(r_b)$ ($\mathcal{B}_{\mathbf{x}}(r)$ denotes the ball centered at \mathbf{x} with radius r_b), and $N_{j,i}^2$ denote that out of $\mathcal{B}_{\mathbf{X}_{BS,i}}(r_b)$. It follows that $N_{j,i} = N_{j,i}^1 + N_{j,i}^2$. Let $|\cdot|$ denote the number of elements in a given set, then the total number of RBs is $N_{RB,i} = |S_{RB,i}^1| + |S_{RB,i}^2|$, and we have

$$|S_{RB,i}^1| = \sum_{\mathbf{X}_{AP,j} \in \Phi_{AP}^{\mathbf{X}_{BS,i}}} 2N_{j,i}^1, \quad |S_{RB,i}^2| = \left(\sum_{\mathbf{X}_{AP,j} \in \Phi_{AP}^{\mathbf{X}_{BS,i}}} 2N_{j,i}^2 \right) / \mu. \quad (1)$$

2.2. Channel Model

Path-loss and fast fading are considered. The received power of a signal transmitted at a distance of d with transmitting power P_T is $P_R(d) = \frac{P_T \cdot H}{l(d)}$, where $l(d)$ is the path-loss function of distance d and H is a random channel gain. A simplified model is

$$l(d) = \kappa_0 \cdot d^\beta, \quad (2)$$

where β is the path-loss exponent, $\kappa_0 = (4\pi/v)^2$, and v is the electromagnetic wavelength. H is an exponentially distributed random variable with mean 1, i.e., Rayleigh fading is considered. Independent fast fading is assumed for different transmitter-receiver pairs. The fast fading between location \mathbf{x}_1 and \mathbf{x}_2 is denoted by $H_{\mathbf{x}_1, \mathbf{x}_2}$.

2.3. Problem Formulation

In the following, a typical BS $\mathbf{X}_{BS,0}$ and its subscribing AP $\mathbf{X}_{AP,0}$ at distance r are considered. Without loss of generality, it is assumed that $\mathbf{X}_{BS,0}$ is located at origin \mathbf{o} and $\mathbf{X}_{AP,0}$ located at $\mathbf{x}_0 = (r, 0)$. To evaluate the multi-hop backhaul, we study the following metrics.

Definition 1 (Forward Progress). *The forward progress (FP) at AP $\mathbf{X}_{AP,0}$, which indicates the distance a packet travels towards the destination at the particular hop, is defined as*

$$FP(\mathbf{X}_{AP,0}) = r - |\mathbf{X}_{AP,R_{0,0}(1)}|. \quad (3)$$

Definition 2 (Hop Distance). *The hop distance at $\mathbf{X}_{AP,0}$ is defined as*

$$L(\mathbf{X}_{AP,0}) = |\mathbf{X}_{AP,0} - \mathbf{X}_{AP,R_{0,0}(1)}|. \quad (4)$$

The downlink throughput is studied. Denote the signal-to-interference-noise-ratio (SINR) of the link from $\mathbf{X}_{AP,R_{0,0}(n+1)}$ to $\mathbf{X}_{AP,R_{0,0}(n)}$ by $\text{SINR}_{0,0}^n$. The single-hop throughput of the link is defined as

$$\mathcal{T}_s(\mathbf{X}_{AP,R_{0,0}(n+1)}, \mathbf{X}_{AP,R_{0,0}(n)}) = R_p \cdot \Pr\{\text{SINR}_{0,0}^n > \theta_{\text{SINR}}\}, \quad (5)$$

where $0 \leq n \leq N_{0,0} - 1$, $R_p = \frac{B}{N_{RB,0}} \log_2(1 + \theta_{\text{SINR}})$ is the information rate per packet, B is the bandwidth of total available spectrum, and θ_{SINR} is the SINR threshold for successful transmission. Note that $\Pr\{\text{SINR}_{0,0}^n > \theta_{\text{SINR}}\}$ is the coverage probability of the link. Since wireless backhaul operates in virtual circuit mode, the throughput of backhaul is bottlenecked by the link with the lowest throughput, i.e.,

$$\mathcal{T}_b(\mathbf{X}_{AP,0}) = \min_{0 \leq n \leq N_{0,0} - 1} \mathcal{T}_s(\mathbf{X}_{AP,R_{0,0}(n+1)}, \mathbf{X}_{AP,R_{0,0}(n)}). \quad (6)$$

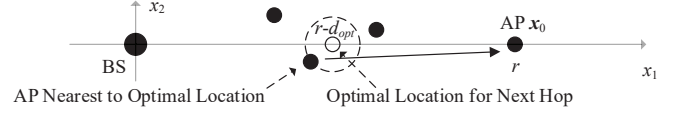


Fig. 1. Illustration of routing selection in $(r, 0)$.

Accordingly, the spectral efficiency of cell $\mathbf{X}_{BS,0}$ is defined as

$$\eta = \frac{1}{B} \sum_{\mathbf{X}_{AP,j} \in \Phi_{AP}^{\mathbf{X}_{BS,0}}} \mathcal{T}_b(\mathbf{X}_{AP,j}). \quad (7)$$

Our goal is to calculate and maximize η with respect to different routing and resource reuse options.

3. THE PROPOSED ROUTING PROTOCOL

In this section, we propose a routing protocol for multi-hop wireless backhaul. Based on the routing protocol, we investigate the bounds for the FP and hop distance, and then calculate the hop number.

3.1. Protocol Design

Ideally, relays should lie on the line segment between the source and the destination to avoid the detour in routing, and their intervals should be equal since the throughput of the path is bottlenecked by the hop with the longest distance. We denote the hop distance by d . Since $N_{j,i}$ is inversely proportional to d , and for a given receiver, the coverage probability of a single hop is a decreasing function with respect to d because of path-loss, there exists an optimal d_{opt} that can maximize (6). Motivated by this observation, we propose that $\mathbf{X}_{AP,0}$ selects its next hop nearest to $\mathbf{X}_{AP,R_{0,0}(1)}^{opt}$, where $\mathbf{X}_{AP,R_{0,0}(1)}^{opt} = (\mathbf{X}_{AP,0} - d_{opt} \frac{\mathbf{X}_{AP,0} - \mathbf{X}_{BS,0}}{|\mathbf{X}_{AP,0} - \mathbf{X}_{BS,0}|})$ is the optimal location of the next hop under the ideal scenario. If $|\mathbf{X}_{AP,0} - \mathbf{X}_{BS,0}| < d_{opt}$, $\mathbf{X}_{AP,0}$ connects to the BS directly. An example of hop selection is illustrated in Fig. 1. The process repeats until $\mathbf{X}_{BS,0}$ is reached. d_{opt} is computed numerically by maximizing the spectral efficiency.

3.2. Approximation of Forward Progress and Hop Distance

In this subsection, we derive the lower bound of FP and the upper bound of hop distance.

Define $E = |\mathbf{X}_{AP,R_{0,0}(1)} - \mathbf{X}_{AP,R_{0,0}(1)}^{opt}|$ as the distance from $\mathbf{X}_{AP,R_{0,0}(1)}$ to its ideal location. For $r > d_{opt}$, in accordance with Slivnyak theorem [9, Theorem 1.4.5], the conditional probability that E is greater than t given $\mathbf{X}_{AP,0}$ located at \mathbf{x}_0 is

$$\Pr\{E > t \mid \mathbf{x}_0\} \leq \Pr\{\Phi_{AP}(\mathcal{B}_{(r-d_{opt},0)}(t)) = \emptyset\} = e^{-\lambda_{AP} \cdot \pi t^2}.$$

If for some τ , $\Pr\{E > \tau \mid \mathbf{x}_0\}$ is less than a small threshold, e.g., 0.01 is applied throughout this paper, $\mathbf{X}_{AP,0}$ can then find its next hop within $\mathcal{B}_{(r-d_{opt},0)}(\tau)$ with a probability as high as 99%. Accordingly, the following approximation is made.

Approximation 1. For an AP at a distance of r from the serving BS, where $r > d_{opt}$, the FP is lower bounded by $FP_{\min} = d_{opt} - \tau$, and the hop distance is upper bounded by $L_{\max} = d_{opt} + \tau$, where $e^{-\lambda_{AP} \cdot \pi \tau^2} = 0.01$.

3.3. Hop Number Calculation

In this subsection, we derive upper bounds for the hop number.

Proposition 1. *Given the AP $\mathbf{X}_{AP,0}$ located at a distance of r from its nearest BS $\mathbf{X}_{BS,0}$, the hop number on the backhaul of $\mathbf{X}_{AP,0}$ is upper bounded by*

$$N_{0,0} \leq \begin{cases} 1, & 0 < r \leq d_{opt}, \\ 1 + \frac{r}{FP_{\min}}, & d_{opt} < r. \end{cases} \quad (8)$$

Proof: For $r \leq d_{opt}$, $\mathbf{X}_{AP,0}$ connects $\mathbf{X}_{BS,0}$ directly. If $r > d_{opt}$, we have $\sum_{n=0}^{N_{0,0}-2} FP(\mathbf{X}_{AP,R_{0,0}(n)}) < r$ because $\sum_{n=0}^{N_{0,0}-1} FP(\mathbf{X}_{AP,R_{0,0}(n)}) = r$, and $|\mathbf{X}_{AP,R_{0,0}(n)}| > d_{opt}$ for $n \leq N_{0,0} - 2$. In accordance with Approximation 1, it follows that $(N_{0,0} - 1) \cdot FP_{\min} < r$, i.e., $N_{0,0} < 1 + \frac{r}{FP_{\min}}$. \square

The upper bound derived in (8) is applicable to any AP given its distance to the nearest BS. Without loss of generality, we denote the upper bound in (8) by $\bar{N}(r)$ to represent the upper bound on the hop number of a generic AP located at a distance r from its nearest BS.

Note that $N_{0,0}^1 = 1$ if $r_b \leq d_{opt}$, and $N_{0,0}^1 = N_{0,0}^1$ if $r \leq r_b$. We make the following approximation on $N_{0,0}^1$ and $N_{0,0}^2$.

Approximation 2. If $r \geq r_b \geq d_{opt}$, the means of $N_{0,0}^1$ and $N_{0,0}^2$ are approximated by

$$\mathbb{E}\{N_{0,0}^1\} \approx \frac{r_b - d_{opt}}{r} (\mathbb{E}\{N_{0,0}\} - 1) + 1, \quad (9)$$

$$\mathbb{E}\{N_{0,0}^2\} \approx \frac{r - r_b + d_{opt}}{r} (\mathbb{E}\{N_{0,0}\} - 1). \quad (10)$$

By replacing the $\mathbb{E}\{N_{0,0}\}$ in (9) and (10) with $\bar{N}(r)$, the upper bounds of $\mathbb{E}\{N_{0,0}^1\}$ and $\mathbb{E}\{N_{0,0}^2\}$ can be obtained. Similar to the case of hop number, these upper bounds are also applicable to any AP given its distance to the nearest BS. Thus, we denote the upper bounds by $\bar{N}^1(r)$ and $\bar{N}^2(r)$ respectively.

Based on Proposition 1 and Approximation 2, the means of $|S_{RB,0}^1|$ and $|S_{RB,0}^2|$ are derived below.

Proposition 2. *The means of $|S_{RB,i}^1|$ and $|S_{RB,i}^2|$ are upper bounded by*

$$\mathbb{E}\{|S_{RB,i}^1|\} \leq \begin{cases} \frac{\lambda_{AP}}{\lambda_{BS} \cdot FP_{\min}} [d_{opt} \cdot (e^{-\pi \lambda_{BS} d_{opt}^2} - e^{-\pi \lambda_{BS} r_b^2}) + \frac{\text{erfc}(\sqrt{\lambda_{BS} \pi} \cdot d_{opt}) - \text{erfc}(\sqrt{\lambda_{BS} \pi} \cdot r_b)}{2\sqrt{\lambda_{BS}}}] + \frac{\lambda_{AP}}{\lambda_{BS}}, & r_b > d_{opt}, \\ \lambda_{AP}/\lambda_{BS}, & 0 < r_b \leq d_{opt}. \end{cases} \quad (11)$$

$$\mathbb{E}\{|S_{RB,i}^2|\} \leq \begin{cases} \frac{\lambda_{AP}}{\lambda_{BS} \cdot FP_{\min} \cdot \mu} [d_{opt} \cdot e^{-\pi \lambda_{BS} r_b^2} + \frac{1}{2\sqrt{\lambda_{BS}}} \cdot \text{erfc}(\sqrt{\lambda_{BS} \pi} \cdot r_b)], & r_b > d_{opt}, \\ \frac{\lambda_{AP}}{\lambda_{BS} \cdot FP_{\min} \cdot \mu} [d_{opt} \cdot e^{-\pi \lambda_{BS} d_{opt}^2} + \frac{1}{2\sqrt{\lambda_{BS}}} \cdot \text{erfc}(\sqrt{\lambda_{BS} \pi} \cdot d_{opt})], & 0 < r_b \leq d_{opt}. \end{cases} \quad (12)$$

Proof: Suppose that all the APs backhaul their data to their nearest BSs. Given a BS $\mathbf{X}_{BS,i}$ and an AP $\mathbf{X}_{AP,j}$ located at a distance of ρ , denote the probability that $\mathbf{X}_{AP,j} \in \Phi_{AP}^{\mathbf{X}_{BS,i}}$ by $p(\rho)$. Applying the Slivnyak theorem, we have

$$p(\rho) = \Pr\{\Phi_{BS}(\mathcal{B}_{\mathbf{x}_0}(\rho)) = 0\} = e^{-\pi \lambda_{BS} \rho^2}. \quad (13)$$

For the typical BS $\mathbf{X}_{BS,0}$ and a given area \mathcal{A} , the summation of $N_{j,0}^1$ of the subscribers of $\mathbf{X}_{BS,0}$ within \mathcal{A} is calculated by

$$|S_{RB,0}^1(\mathcal{A})| = \sum_{\mathbf{X}_{AP,j} \in \Phi_{AP} \cap \mathcal{A}} \mathbb{1}(\mathbf{X}_{AP,j} \in \Phi_{AP}^{\mathbf{X}_{BS,0}}) \cdot N_{j,0}^1. \quad (14)$$

The expectation of (14) is

$$\begin{aligned} & \mathbb{E}\{|S_{RB,0}^1(\mathcal{A})|\} \\ &= \sum_n \Pr\{\Phi_{AP}(\mathcal{A}) = n\} \mathbb{E}\{|S_{RB,0}^1(\mathcal{A})| \mid \Phi_{AP}(\mathcal{A}) = n\} \\ &\leq \lambda_{AP} \cdot \int_{\mathcal{A}} p(|\mathbf{x}|) \cdot \bar{N}^1(|\mathbf{x}|) d\mathbf{x}. \end{aligned}$$

So as $\mathcal{A} \rightarrow \mathbb{R}^2$, the expectation of $|S_{RB,0}^1|$ is upper bounded by

$$\mathbb{E}\{|S_{RB,0}^1|\} \leq 2\pi \lambda_{AP} \cdot \int_0^\infty \rho \cdot p(\rho) \cdot \bar{N}^1(\rho) d\rho,$$

whose results are given in (11). The upper bounds of $|S_{RB,0}^2|$ can be derived similarly. Because of the stationarity, the results are applicable to any BS. \square

It can be observed that $\mathbb{E}\{|S_{RB,i}^1|\}$ and $\mathbb{E}\{|S_{RB,i}^2|\}$ are constant for r_b over $(0, d_{opt}]$, so we only consider the case with $d_{opt} \leq r_b$. Let $\bar{|S_{RB,i}^1|}$ and $\bar{|S_{RB,i}^2|}$ denote the upper bounds of $\mathbb{E}\{|S_{RB,i}^1|\}$ and $\mathbb{E}\{|S_{RB,i}^2|\}$, respectively. For simplicity, hereinafter, we replace random variables $|S_{RB,i}^1|$ and $|S_{RB,i}^2|$ by $\bar{|S_{RB,i}^1|}$ and $\bar{|S_{RB,i}^2|}$.

4. INTERFERENCE MODELING

In this section, we investigate both intra-cell interference I_i and inter-cell interference I_o at the typical AP $\mathbf{X}_{AP,0}$. Without loss of generality, we assume that $\mathbf{X}_{AP,0}$ receives signals over RB 1. For simplicity of notation, in this section, we set transmit power $P_T = 1$.

4.1. Intra-cell interference

First note that $I_i \neq 0$ only if $r > r_b$, $\mu \neq 1$ and RB 1 $\in S_{RB,0}^2$. Since APs closer to the BS are likely to generate more interference due to a heavier load of relaying, the distribution of interferers is not homogeneous. We make the following approximations to render the interference analysis tractable.

Approximation 3. The distribution of $\Phi_{AP}^{\mathbf{X}_{BS,0}}$ is approximated as an independent thinning [9, Proposition 1.3.5] of Φ_{AP} with retention function $p(\rho)$ in (13), i.e., $\Phi_{AP}^{\mathbf{X}_{BS,0}}$ is a PPP with intensity $\lambda_{AP}^{\mathbf{X}_{BS,0}}(\rho) = e^{-\pi \lambda_{BS} \rho^2} \cdot \lambda_{AP}$ at distance ρ from origin.

Approximation 4. For any given RB in $S_{RB,0}^2$ and any AP $\mathbf{X}_{AP,j} \in \Phi_{AP}^{\mathbf{X}_{BS,0}}$ at a distance ρ from $\mathbf{X}_{BS,0}$, the probability that the RB will be allocated to the backhaul of $\mathbf{X}_{AP,j}$ is $\bar{N}_{j,0}^2(\rho)/\bar{|S_{RB,0}^2|}$.

Approximation 5. Define $\Phi_{AP,bb,1}^{\mathbf{X}_{BS,0}} = \{\mathbf{X}_{AP,j} \mid \mathbf{X}_{AP,j} \in \Phi_{AP}^{\mathbf{X}_{BS,0}}, RB_{j,0}^1(n) = 1 \text{ for some } n\}$. For a given $\mathbf{X}_{AP,j} \in \Phi_{AP,bb,1}^{\mathbf{X}_{BS,0}}$, let $\mathbf{X}_{AP,R_{j,0}^1}$ denote the hop on its backhaul transmitting over RB 1. $|\mathbf{X}_{AP,R_{j,0}^1}|$ is uniformly distributed over $(r_b - d_{opt}, |\mathbf{X}_{AP,j}|)$.

Based on Approximation 3 and 4, $\Phi_{AP,bb,1}^{\mathbf{X}_{BS,0}}$ is a PPP with intensity $\lambda_{AP,bb,1}(\rho) = \frac{\bar{N}^2(\rho)}{\bar{|S_{RB,0}^2|}} \cdot \lambda_{AP} \cdot e^{-\pi \lambda_{BS} \rho^2}$ with respect to the distance ρ . Define $\Phi_{AP,1}^{\mathbf{X}_{BS,0}}$ as the set of APs in $\Phi_{AP}^{\mathbf{X}_{BS,0}}$ transmitting over RB 1.

Based on Approximation 5 and in accordance with the displacement theorem [9, Theorem 1.3.9], $\Phi_{AP,1}^{X_{BS,0}}$ is still a PPP, and its intensity is

$$\lambda_{AP,1}(\rho) = \begin{cases} 0, & \rho \leq r_b - d_{opt}, \\ \frac{\lambda_{AP} \cdot e^{-\pi \cdot \lambda_{BS} \cdot r_b^2}}{|S_{RB}^2| \cdot FP_{min} \cdot \lambda_{BS} \cdot 2\pi \cdot \rho}, & r_b - d_{opt} < \rho \leq r_b, \\ \frac{\lambda_{AP} \cdot e^{-\pi \cdot \lambda_{BS} \cdot \rho^2}}{|S_{RB}^2| \cdot FP_{min} \cdot \lambda_{BS} \cdot 2\pi \cdot \rho}, & r_b < \rho. \end{cases} \quad (15)$$

4.2. Inter-cell interference

The inter-cell interference is generated by the BSs/APs in other cells transmitting over RB 1. We assume that the allocation of RBs to $S_{RB,i}^1$ and $S_{RB,i}^2$ are independent across different cells. Note that an arbitrary RB would be reused

$$\bar{\mu} = \frac{|S_{RB}^1| + \mu \cdot |S_{RB}^2|}{|S_{RB}^1| + |S_{RB}^2|} \quad (16)$$

times per cell on average. As a result, we make the following approximation.

Approximation 6. The distribution of out-of-cell interferers is approximated as a homogeneous PPP with intensity $\bar{\mu} \cdot \lambda_{BS}$ over \mathbb{R}^2 .

Given the hop distance d , the coverage probability of the virtual link from $X_{AP,R0,0(1)}$ to $X_{AP,0}$ can be calculated by making use of the Rayleigh fading assumption [4], which involves the calculation of the LT of the interference [9, Proposition 2.2.4], and the throughput can be calculated accordingly. Since d is upper bounded by L_{max} , the lower bound of the throughput of a single hop can be obtained by setting $d = L_{max}$. Then, the throughput of the backhaul associating to $X_{AP,0}$ is lower bounded by that of the single hop with the lowest throughput within $\mathcal{B}_o(|X_{AP,0}|)$. The spectral efficiency can be calculated by treating the throughput of each AP as an independent mark and making use of the property of independent marked Point Process (IMPP) [9, Corollary 2.1.2].

5. NUMERICAL RESULTS

In this section, we demonstrate the performance of multi-hop backhaul via numerical results. Unless otherwise mentioned, we use the following parameters: carrier frequency $f_c = 1$ GHz, $\lambda_{BS} = 10^{-6}/m^2$, $\lambda_{AP} = 1 \times 10^{-2}/m^2$, $\beta = 4$, $\theta_{SINR} = 10$ dB, $N_0 = -165$ dBm/Hz, $B = 10$ MHz. To make the comparison fair, the transmit power is normalized by the average resource reuse factor, $P_T/\bar{\mu}$, to ensure equal power consumption for different μ . The optimal d_{opt} and r_b is obtained by maximizing the spectral efficiency through Sequential Quadratic Programming (SQP) in Matlab.

Table 1 lists the optimized parameters for different power and resource reuse settings. *It is observed that:* (1) The optimal d_{opt} decreases as transmit power increases. This is because a transmitter with higher power could potentially cover a wider area. (2) The optimal d_{opt} decreases as the resource reuse factor, μ , increases because the intra-cell interference could become more serious with heavier resource reuse, which would limit the transmit distance per hop. (3) For $\mu \neq 1$, the optimal r_b always equals d_{opt} . That is, except for the RBs occupied by the BS, all other RBs should be reused within the cell, indicating that it is not efficient to sacrifice the bandwidth for exchange of a lower intra cell interference.

Fig. 2 shows the spectral efficiency for different power and resource reuse settings. *It is observed that:* resource reuse can significantly improve the spectral efficiency, but spectral efficiency is not

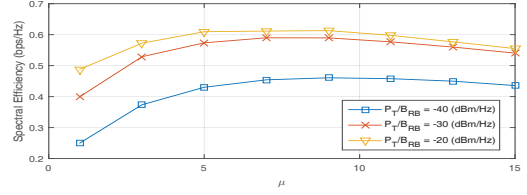


Fig. 2. Spectral efficiency for different μ and P_T/B_{RB}

Table 1. The optimal d_{opt} and r_b for different P_T/B_{RB} and μ

P_T/B_{RB}	-40 dBm/Hz			-30 dBm/Hz		
μ (m)	1	7	13	1	7	13
d_{opt} (m)	71.28	39.14	30.53	115.47	55.79	39.03
r_b (m)	-	39.14	30.53	-	55.79	39.03

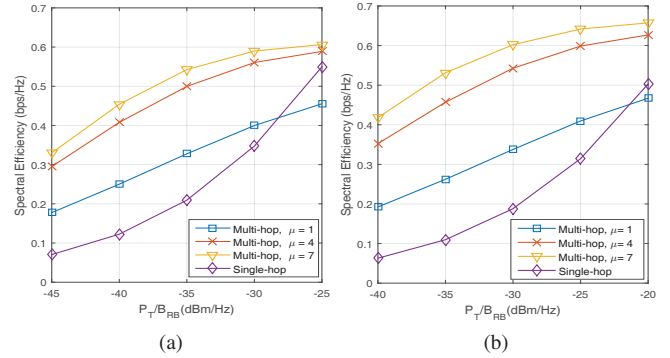


Fig. 3. Comparison of spectral efficiency for (a) $\lambda_{BS} = 10^{-6}/m^2$ and (b) $\lambda_{BS} = 5 \times 10^{-7}/m^2$.

monotonically increasing with respect to μ , because the increased number of RBs for smaller FP offsets the expanded bandwidth per RB introduced by a larger μ .

Fig. 3 shows the spectral efficiency of the proposed multi-hop wireless backhaul in comparison with the single-hop one. *It is observed that:* when the transmit power is low or the BSs are sparse, the proposed scheme obtains a much higher spectral efficiency than the single-hop wireless backhaul, because the APs far away cannot reach the BS directly, which necessitates a relay scheme. However, if the transmit power is high, the single-hop scheme is preferred. This is because once the single hop link can satisfy the SINR requirement, the advantage of relaying vanishes, and the extra bandwidth required in the multi-hop scheme would limit its throughput.

6. CONCLUSIONS

In this paper, we studied the multi-hop wireless backhaul in heterogeneous cellular networks. We proposed a routing protocol for throughput optimization, and a tractable model of intra-cell interferers by utilizing stochastic geometry. Numerical results showed that the proposed scheme can considerably improve the spectral efficiency of cellular network over the single-hop backhaul, under the scenarios where the transmit power is limited or the BSs are sparsely deployed.

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