PUTTING STATISTICS INTO THE STATISTICAL ENERGY ANALYSIS OF AUTOMOTIVE VEHICLES

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ABSTRACT
Sound and vibration transmission modeling methods are important to the design process for high quality automotive vehicles. Statistical Energy Analysis (SEA) is an emerging design tool for the automotive industry that was initially developed in the 1960’s to estimate root-mean-square sound and vibration levels in structures and interior spaces. Although developed to estimate statistical mean values, automotive design application of SEA needs the additional ability to predict statistical variances of the predicted mean values of sound and vibration. This analytical ability would allow analysis of vehicle sound and vibration response sensitivity to changes in vehicle design specifications and their statistical distributions.

This paper will present an algorithm to extend the design application of the SEA method through prediction of the variances of RMS responses of vibro-acoustic automobile structures and interior spaces from variances in SEA automotive model physical parameters. The variance analysis is applied to both a simple, complete illustrative example and a more complex automotive vehicle example. Example variance results are verified through comparison with a Monte Carlo test of 2,000 SEA responses whose physical parameters were given Gaussian distributions with means at design values. Analytical predictions of the response statistics agree with the statistics generated by the Monte Carlo method but only require about 1/500 of the computational effort.

INTRODUCTION
Statistical Energy Analysis (SEA) is of particular interest to the automotive industry because it can predict sound and vibration levels at higher frequencies than differential equation based analytical methods. SEA has been successfully employed by the aerospace industry (Lyon, R.H., 1975, Jacobs, E.W., et al., 1989) and the maritime industry (Jenson, J.O., 1979). The analytical foundation of SEA is the concept that the vibration and sound energy of dynamic systems is shared equally between modes over narrow band of frequency (Lyon, R.H., 1975). For automotive vehicles, this concept makes SEA most accurate at frequencies above 300-500 Hz, where the number of modes in octave bands is high enough to allow averaging of the energy between the modes. As an example, a typical automotive interior has approximately 27 modes per Hz. at 5,000 Hz. Differential equation methods like the Finite Element Method have difficulty maintaining accuracy due to the same high modal density at these frequencies (Remington, P.J. and Manning, J.E., 1975, Lu, L.K.H., et al., 1983).

Design of quality vehicles requires analytical methods that can accurately predict both the expected performance of designs and the variance of vehicle performance about the expected performance. Design analysis is needed for two purposes, the evaluation of existing vehicle designs and the development of specifications for new designs. Existing vehicle designs are evaluated to predict typical vehicle vibration and acoustic response and the probability that a vehicle will perform within specified performance specifications. During vehicle design development, subsystem specifications are used to develop predictions of expected system performance and variance. Both expected value and variance of SEA models are required to predict confidence levels in both classes of design analysis.

STATISTICAL ENERGY ANALYSIS
Statistical Energy Analysis can be illustrated with the simple automotive vehicle idealized as a three element SEA model in Fig. 1. The figure represents vibration and acoustic energy storage by the energies, $E_1$, $E_2$ and $E_3$ in the three SEA element blocks. Vibration and acoustic power flow between the three elements transmitted into the interior through the vehicle structure and flanking paths is shown by the arrows labeled $P_{12}$, $P_{13}$, and $P_{23}$ where the subscripts refer to the elements connected. The external power input $P_1$ comes from engine surface vibration in the engine compartment. Power dissipated by vehicle damping
and acoustic treatment is represented by the three arrows $P_{diss1}$, $P_{diss2}$, and $P_{diss3}$.

SEA makes the fundamental assumption that, within narrow frequency bands, the energies in all independent modes equalize at steady state. The total energy in each element, $E_i$, in a frequency band centered at $f$ is derived from the product of the element's modal energy, $\epsilon_i$, and the element's modal density, $\eta_i$, at that frequency.

$$E_i = E_i(f, x) = \eta_i(f) \xi_i$$  \hspace{1cm} (1)

The total element energy can be used to compute the expected value of mean-rms velocity for structural elements,

$$E_i = \eta_i \left( \mathbf{\xi} \right)$$  \hspace{1cm} (2)

where: $m_i$ is the mass of the structure

$\left\{ \mathbf{\xi} \right\}$ is the expected squared velocity or the expected value of mean-rms sound pressure in acoustic spaces.

$$E_i = \frac{V_i}{c^2} \left( \mathbf{\xi} \right)$$  \hspace{1cm} (3)

where: $V_i$ is the volume of the space

$P$ is the density of the medium

$c$ is the speed of sound

$\left\{ \mathbf{\xi} \right\}$ is rms sound pressure level

The SEA model equations derived from conservation of power and equal modal energies take the form of simultaneous, frequency-dependent, algebraic equations. For a $K$ element model, they can be written as:

$$Ne = \left( \frac{1}{2 \pi f} \right) P$$  \hspace{1cm} (4)

where: $\mathbf{\xi} = [\xi_1, ..., \xi_K]^T$, SEA element mean-rms modal energy vector

$$N = N(\eta, \eta_y, \eta_n),$$ SEA coefficient matrix

$$\eta_i = \eta_i(f, x),$$ element internal loss factors

$$\eta_y = \eta_y(f, x),$$ coupling loss factor between elements $i$ and $j$

$$\eta_n = \eta_n(f, x),$$ element modal density

$f$ is the SEA analysis band center frequency

$x = [x_1, ..., x_M]^T$, SEA physical parameters vector and

$$P = [P_1(f), ..., P_K(f)]^T$$, input power vector.

The SEA coefficient matrix, $N = N(\eta, \eta_y, \eta_n)$, is a symmetric matrix constructed by SEA parameters modal density $\eta_n$, internal loss factor $\eta_i$ and coupling loss factor $\eta_y$.

$$N = \begin{bmatrix}
\eta_i \eta_{i1} & \ldots & \eta_i \eta_{iK} \\
\ldots & \ddots & \ldots \\
\eta_i \eta_{K1} & \ldots & \eta_i \eta_{KK}
\end{bmatrix}$$  \hspace{1cm} (5)

Current SEA modeling software provides RMS response predictions for such an SEA automotive vehicle model. Early SEA development by Lyons (1975) provided variance analysis for a few simple example systems and the response variance for randomly varying resonant frequencies was studied by Delong (1985), however, no general approach to the variance analysis problem for design parameters is currently available in literature. The work discussed here extends SEA to include variance calculations.

**SEA Response Variance Analysis**

Response variance analysis gives the statistical distribution of predicted SEA energy levels about their mean values. Conventional SEA predicts the expected (RMS) values of SEA energies in the form of frequency spectra. Random distributions of physical parameter values lead to variance of predicted SEA energies about their expected frequency spectra values. These variances in SEA responses are implicitly, non-linearly, associated with variances of physical parameters via the SEA equation (4). This implicit variance relationship is derived through nonlinear definitions of the SEA parameters such as modal density, internal loss factor and coupling loss factor. SEA response statistics are dependent on the statistics of the model's
physical parameters in these definitions. We will derive a linear approximate variance expression for SEA responses about means in terms of physical parameters through a Taylor expansion.

The SEA response deviation about its mean-rms energy can be approximated by first order Taylor expansion over the \( M \) physical parameters and \( K \) SEA element inputs.

\[
\Delta E_i = E_i - \bar{E}_i = \sum_{m=1}^{M} \frac{\partial E_i}{\partial x_m} \Delta x_m + \sum_{k=1}^{K} \frac{\partial E_i}{\partial P_k} \Delta P_k
\]

(6)

The partial derivatives of the \( i \)th element's energy response about are evaluated at the nominal physical parameters \( x = \bar{x} \) and nominal power inputs \( P = \bar{P} \).

Taking the variance (Clifford, A.A., 1973; Oh, H.L., 1987) of (6) yields the variance of SEA responses, \( \sigma^2(E_i) \), in terms of the variances of \( M \) physical parameters and \( K \) input powers.

\[
\sigma^2(E_i) = \sum_{m=1}^{M} \left( \frac{\partial E_i}{\partial x_m} \right)^2 \sigma^2(x_m) + \sum_{k=1}^{K} \left( \frac{\partial E_i}{\partial P_k} \right)^2 \sigma^2(P_k)
\]

(7)

where: \( \sigma^2(E_i) \) is the variance of total energy response in the \( i \)th element,

\( \sigma^2(x_m) \) is the variance of the \( m \)th physical parameter, and

\( \sigma^2(P_k) \) is the variance of the \( k \)th power input.

The SEA response derivatives are derived with the chain rule by using (1).

\[
\frac{\partial E_i}{\partial x_m} = n_i \frac{\partial E_i}{\partial \bar{x}_m}
\]

(8a)

\[
\frac{\partial E_i}{\partial P_k} = n_i \frac{\partial E_i}{\partial \bar{P}_k}
\]

(8b)

Note that (7) is derived assuming independent physical parameter statistics without making any assumptions on probability density functions of these parameters, and can be used to predict the probability that the vehicle will perform within specified performance specifications. It can also be used to analyze SEA model response variances for an existing design and to develop required specifications for a new design.

Partial derivatives of modal energies with respect to physical parameters needed to calculate SEA response variances can be derived by taking derivatives of SEA equation (6) with respect to physical parameter \( x_m \) and noting that the input power powers are independent of the element physical parameters, \( \partial P/\partial x_m = 0 \).

This analysis yields a relation between the partial derivative of the modal energies and the partial derivative of the SEA coefficient matrix, \( N \).

\[
\frac{\partial N}{\partial x_m} = -N^{-1} \frac{\partial N}{\partial x_m} * N
\]

(9)

Derivatives of the SEA coefficient matrix \( N \) with respect to a physical parameter \( x_m \) can be obtained by the chain rule.

\[
\frac{\partial N}{\partial x_m} = \sum_{k=1}^{K} \frac{\partial N}{\partial x_m} \frac{\partial x_m}{\partial x_m} + \sum_{i=1}^{I} \frac{\partial N}{\partial x_m} \frac{\partial x_m}{\partial x_m} + \sum_{j=1}^{J} \frac{\partial N}{\partial x_m} \frac{\partial x_m}{\partial x_m}
\]

(10)

The modal densities, internal loss factors, and coupling loss factors appear explicitly in the SEA coefficient matrix \( N \). Modal density, \( n_i \), and internal loss factor, \( \eta_i, \) in the \( i \)th element are a function of the physical parameters of that element only. The coupling loss factors, \( \eta_{ij} \), are a function only of the \( i \)th and \( j \)th elements' physical parameters when those elements are coupled in the SEA model. With these constraints, most of the partial derivatives in (10) become zero when it is evaluated.

Derivatives of the SEA coefficient matrix \( N \) with respect to modal densities \( n_i \) are derived from (5) by noting that a modal density, \( n_i \), appears only in the \( i \)th row, the \( i \)th column and the diagonal of the SEA coefficient matrix.

\[
\frac{\partial N}{\partial n_i} = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & (n_i + \sum_{j=i+1}^{I} \eta_{ij}) & \cdots & -n_i \\ 0 & \cdots & -n_i & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & -n_i & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \\ \end{bmatrix}
\]

(11)

Derivatives of the SEA coefficient matrix with respect to internal loss factors \( \eta_{ij} \) are derived from (5) by observing that an internal loss factor, \( \eta_{ij} \), appears only in the \( ij \)th diagonal element.

\[
\frac{\partial N}{\partial \eta_{ij}} = \begin{bmatrix} \delta_{ij} & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \delta_{ij} & \cdots & \delta_{ij} & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \delta_{ij} & \cdots & \delta_{ij} & \cdots & \delta_{ij} \\ \end{bmatrix}
\]

(12)

Derivatives of the SEA coefficient matrix (5) with respect to coupling loss factors, \( \eta_{ij} \), makes use of the observation that the coupling loss factor, \( \eta_{ij} \), appears only at the intersects of the \( ij \)th
and jth rows and columns of the SEA coefficient matrix (5). Here we assume that \( j > i \) to yield the result below.

\[
\frac{\partial N}{\partial \eta_j} = \begin{bmatrix} \frac{\partial N_{km}}{\partial \eta_j} \end{bmatrix}
\]

\[
= \begin{cases} \eta_j, & k = m 	ext{ for } k = j \\ -\eta_j, & k = i, m \neq k, j, m \neq i \\ 0, & \text{otherwise} \end{cases}
\]

(13)

The derivative of the modal energy vector, \( \mathbf{e} \), with respect to the element power inputs, \( P_i \), are found by observing that the partial derivative of the power input vector, \( P \), is non-zero in the jth row only.

\[
\frac{\partial \mathbf{e}}{\partial P_j} = \frac{1}{\omega} \mathbf{N}^{-1} 
\]

(14)

The terms in the derivatives of SEA coefficients with respect to physical parameters are dependent on specific element and coupling factor definitions and can be simple or complicated. Some terms such as the loss coefficients for structural and acoustic elements such as flat plates and 3-dimensional acoustic volumes are simple analytical functions of SEA parameters, \( \chi \), and analytical expressions for their derivatives are found easily. Other terms like the coupling coefficients between beam and plates have complex and/or discontinuous expressions and their derivatives may need to be approximated numerically in the vicinity of the parameter mean values. The first example below will present the analytical process in detail because the SEA model is simple enough to allow a complete presentation of the analysis. To demonstrate the effectiveness of the analysis on a more realistic and complex automotive model, only the model perturbations and the results derived will be presented. The purpose of this study is an initial evaluation of the ability to use the expressions derived above to find estimates of vibro-acoustic response variances.

**EXAM PLES AND MONTE CARLO VALIDATION**

Two examples and a Monte Carlo modeling procedure are used here to validate the variance analysis. A set of two thousand (2,000) Gaussian distributed parameter values are first generated to simulate a production process. The mean and variances of these parameters are used in the variance analysis to predict the mean and variance of SEA response energies. The two thousand parameter values are then used in two thousand separate SEA analyses to compute 2,000 actual SEA responses. The mean and variance of these actual SEA responses are then compared with the variance analysis predictions of mean and variance. The first example is a simple conceptual model (Fig. 2) to illustrate the details of the analytical development and Monte Carlo test verification. This SEA model has the topology similar to that shown in Fig. 1. The SEA model is two identical cubic spaces separated by a flat square aluminum plate and includes a flanking path between the two acoustic spaces (Table 1, Appendix). Acoustic power is input to the first cube and 5,000 Hz is taken as the analysis band center frequency. In the variance analysis, the plate thickness \( h \) is chosen to be non-stationary, \( x_i = h \) and we calculate the variance of total energy response in the second acoustic space, \( \sigma(E_2) \). A Gaussian distribution of 2,000 plate thicknesses with mean value of 7.5 x 10^{-3} m and standard deviation of 5% of the mean, \( \sigma_{x_i} = 3.75 \times 10^{-4} \) (mms), is generated from a MathLab software call. Note that the actual standard deviation of the data sample generated does not exactly equal the requested value for the sample. The histogram of 2,000 Gaussian distributed plate thicknesses is shown in Fig. 3.

The variance of the second acoustic space response (7) is dependent only on the variance of the plate thickness because the variances of the input powers are zero.

\[
\sigma^2(E_2) = \left[ \frac{\partial E_2}{\partial x_i} \right]_{P_{i} = \bar{P}} \sigma^2(x_i)
\]

(15)

The derivative of total energy response with respect to \( x_i \) incudes only the first term in (8a) because modal density in the second acoustic space is independent of the plate thickness.

\[
\frac{\partial E_2}{\partial x_i} = n_{21} \frac{\partial E_1}{\partial x_i}
\]

(16)

The derivative of the SEA coefficient matrix \( \mathbf{N} \) with respect to plate thickness is needed (9) in order to compute the derivative of modal response with respect to plate thickness. Since the plate thickness is associated only with the plate modal density, \( n_{21} \), coupling loss factors \( \eta_{22} \) and \( \eta_{23} \) (10) includes only three terms.
FIGURE 3: PLATE THICKNESS HISTOGRAM AND CORRESPONDING GAUSSIAN PROBABILITY DENSITY

\[
\frac{\partial N}{\partial \eta_3} = \frac{\partial N}{\partial \eta_2} + \frac{\partial N}{\partial \eta_1} + \frac{\partial N}{\partial \eta_2} \frac{\partial \eta_2}{\partial \eta_1} (17)
\]

The terms, \( \frac{\partial N}{\partial \eta_1} \), \( \frac{\partial N}{\partial \eta_2} \), and \( \frac{\partial N}{\partial \eta_3} \), can be obtained by using (11) and (13). The derivatives with respect to \( \eta_1 \) at 5,000 Hz of modal density \( \eta_1 \), as well as coupling loss factors \( \eta_2 \) and \( \eta_3 \), can be derived from their definitions (Appendix).

\[
\frac{\partial N}{\partial \eta_1} = \frac{\sqrt{2\pi}}{\alpha^{2}c_1} (18)
\]

\[
\frac{\partial \eta_2}{\partial \eta_1} = \frac{\partial \eta_2}{\partial \eta_1} \left( \frac{n_2 \eta_2}{n_1} \right) = \frac{\partial \eta_2}{\partial \eta_1} \left( \frac{n_2 \eta_2}{n_1} \right) + \frac{\partial \eta_2}{\partial \eta_1} (20)
\]

The above analysis gives \( \frac{\partial N}{\partial \eta_1} \bigg|_{\eta_1=5} = 4.9 \times 10^{-4} \) and \( \sigma(\eta_2) = 5 \times 10^{-7} \). Monte Carlo verification of the above variance analysis is started by computing 2,000 SEA model responses from 2,000 separate models, one for each of the 2,000 thicknesses used in the SEA model. The standard deviation of the 2,000 values for total energy response in the second cube, \( \varepsilon_{MC} \), from the Monte Carlo test is \( \sigma(\varepsilon_{MC}) = 1.4 \times 10^{-7} \). The histogram for the 2,000 total energy responses computed for the second acoustic space is plotted in Fig. 4 along with the predicted distribution (solid line). In order to further compare the analytical results with the Monte Carlo results, mean value and variance for the actual model responses in the second acoustic space, \( \varepsilon_{MC} \), were used to plot the Gaussian probability density function (broken line) for the 2,000 model analyses.

The variance analysis probability density function result (solid line) agrees very well with the probability density function computed for the 2,000 SEA model Monte Carlo test. The linear analytical standard deviation prediction has an error that is less than 5% of the actual standard deviation for the 2,000 models computed from Monte Carlo test results. This error can be

FIGURE 4: GAUSSIAN ANALYTICAL PROBABILITY DENSITY FUNCTION (PDF) PREDICTION FOR ACOUSTIC PRESSURE IN SPACE #2 OF SEA EXAMPLE 1 COMPARED WITH HISTOGRAM AND PDF FOR 2,000 MODEL MONTE CARLO TEST RESULTS
Vibration Isolation, Acoustics, and Damping in Mechanical Systems

FIGURE 5: SEA AUTOMOTIVE VEHICLE MODEL WITH BODY PANEL, FLOOR PANEL, HOOD, FRAME, FRONT OF DASH, UNDER CAR, OVER CAR, ENGINE COMPARTMENT, INTERIOR AS ACOUSTIC SPACE.

FIGURE 6: SEA AUTOMOTIVE VEHICLE MODEL ELEMENT COUPLING CHART

partially attributed to the population of random physical parameter used in Monte Carlo tests. Fig. 3 shows that the thickness population is not large enough (2,000 models) for the random generator used in the computation to approach a Gaussian distribution accurately. Increasing the number of models generated in the Monte Carlo tests would have required a prohibitive amount of computational effort. The remaining error can be attributed to the effects of non-linearities in the parameters on predicted variance and will be discussed in the second example.

The second example is an automotive vehicle SEA model (Fig. 5). The vehicle is modeled by 10 elements and 13 couplings (Fig. 6). Structural vibration energy storage models the "Engine" block, "Frame", "Hood", Front of "Dash", "Floorpan", and "Body" panels. Acoustic pressure energy storage models the engine compartment ("EngComp"), under car volume ("UndCarVol"), over car volume ("OvrCarVol"), and interior ("Interior"). Model input power is applied to the "Engine" block only.

To demonstrate the variance analysis, the floor panel thickness is chosen to be non-stationary and analysis of the vehicle interior sound level will be conducted at a band center frequency of 5,000 Hz. Two thousand (2,000) of the Gaussianly distributed floor panel thicknesses are generated with mean 7.50 x 10^-4 (m) and 5% standard deviation 0.38 x 10^-4 (m). The floor panel thickness histogram and associated ideal Gaussian probability density function are presented in Fig. 7. The mean and the variance of the floorpan data generated are then used in the variance analysis to compute predictions of mean and variance of the SEA energy responses. These predictions used formulas developed in this paper and evaluated the derivative of the interior total energy responses with respect to the floor panel thickness at the nominal system parameters. The predicted variance of the interior total energy response at 5,000 Hz. from floor panel thickness variance is 2.38 x 10^-11 (in/ch). After computing 2,000 of SEA total energy responses in the interior at 5,000 Hz by our Monte Carlo procedure, actual model interior acoustic energy responses were found to have a variance of 2.79 x 10^-11 (in/ch). The predicted and actual variance differ by less than 2%. The analytical variance prediction requires only about 1/500 of the computational effort (flops) that required to compute the 2,000 models responses. The histogram of the interior total energy responses and the Gaussian probability density functions from Monte Carlo test and analytical variance analysis at 5,000 Hz. are shown in the Fig. 8. The results show that the linear variance approximations are effective to a complicated automotive vehicle SEA model.

The effect of non-linearities in the SEA coefficient computations on the accuracy of the Taylor expansion was evaluated by running 8 different Monte Carlo validations for differing floorpan variances (off mean value). 2,000 of the physical parameter are generated from the Gaussian distributions for each of the standard deviation choices, 2,000 total interior energy responses (in Joules) are computed for each of the cases and the standard deviations for the responses are computed from Monte Carlo test runs and the analytical standard deviations from linear variance analysis are computed for each of the cases in Fig. 9.

The linear variance analysis prediction for a standard deviation of 15% of floor panel thickness has only a 6% error relative to the actual Monte Carlo test results which corresponds to only about 0.7% of the mean vehicle interior sound energy of 7.33 x 10^-11 Joules computed for the models. Note that a 5-15% floor panel thickness standard deviation may well be greater than acceptable in any actual production process and is used here for discussion purposes only. The variance linear approximation is quite effective and robust to physical parameter variations.
CONCLUSIONS

The analytical variance theory put forth in this paper can accurately predict the SEA model response statistics. The linear approximation is shown to be robust over a broad range of the physical parameter variations tested and the effectiveness of the analytical algorithm is validated by two examples. The algorithm can be applied to evaluate the robustness of existing vehicle designs to variations of design parameters and to develop specifications for new designs. In particular, the algorithm can also be used in the early design stages for a prototype. The accurately predicted variances of SEA model with expected values can be used to predict confidence levels in the design analyses. With these features and applications, the variance theory on SEA vibro-acoustic responses provides a powerful design tool for quality automotive vehicle designs. Designers can acquire quantitative information on the relationship between the design SEA responses and the design physical parameters affecting the mean-rms performance and the variability in the design performance caused by the variability in the design physical parameters. This will allow designers to develop specifications to achieve optimal and/or ultimate robust designs.

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REFERENCES


FIGURE 8: HISTOGRAM AND GAUSSIAN PROBABILITY DENSITIES FROM ANALYTICAL AND MONTE CARLO TESTS FOR STANDARD DEVIATION 5% OF FLOOR PANEL THICKNESS AT 5,000 HZ.

FIGURE 9: COMPARISON OF STANDARD DEVIATIONS OF RESPONSE ENERGY IN THE VEHICLE INTERIOR AT 5,000 HZ FROM ANALYTICAL PREDICTION AND MONTE CARLO TEST DUE TO DEVIATION IN FLOOR PANEL THICKNESS.

FIGURE 10: RELATIONSHIP BETWEEN VIBRATION LEVEL AND FLOOR PANEL THICKNESS.
TABLE 1: GEOMETRIC AND MATERIAL PROPERTIES FOR EXAMPLE 1

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Wavespeed in Plate (m/s)</td>
<td>5181</td>
</tr>
<tr>
<td>Speed of Sound in Air (m/s)</td>
<td>344</td>
</tr>
<tr>
<td>Plate Thickness (m)</td>
<td>7.5 x 10^{-3}</td>
</tr>
<tr>
<td>Length of the Cube (m)</td>
<td>0.500</td>
</tr>
<tr>
<td>Air Density (kg/m^3)</td>
<td>1.244</td>
</tr>
<tr>
<td>Aluminum Density (kg/m^3)</td>
<td>2790</td>
</tr>
<tr>
<td>Reverberation Time of Volume (sec)</td>
<td>1.50</td>
</tr>
<tr>
<td>Power Input in Element 1 (Watts)</td>
<td>1.48 x 10^{-2}</td>
</tr>
</tbody>
</table>

2. Coupling Loss Factors

The coupling loss factor from a panel to an acoustic space is given:

\[ \eta_{ls} = \eta_{ld} \]

\[ \eta_{ld} = \frac{n}{2\pi f_{c}} \left( \frac{2\pi P_{c}}{2\pi f_{c}} \right)^{1/2} \left( \frac{f_{c}}{f} \right)^{1/2} \quad f < f_{c} \]

\[ \eta_{ld} = 0 \quad f > f_{c} \]  

(A3)

where:
- \( \rho \) is density of media
- \( c \) is the speed of sound in the media
- \( A \) is the area of the plate
- \( k \) is the thickness of the plate
- \( \rho \) is the longitudinal wave speed in the plate
- \( f_c \) is the critical frequency
- \( \lambda_c = \frac{c}{f_c} \) is the wavelength of sound at the critical frequency
- \( \beta \) is related to edge fixation:

APPENDIX

Example 1 uses two identical cubic volumes filled with air and separated by an aluminum plate. Formulas used to calculate the SEA parameter derivatives are listed below (Lyons, 1975):

1. Modal Density:

For structure plate:

\[ n_s = \frac{\sqrt{\pi k}}{c_l} \]  

(A1)

where:
- \( A \) is the area of the plate
- \( k \) is the thickness of the plate
- \( c_l \) is the longitudinal wave speed in the plate

For the acoustic volumes:

\[ n_a = n_s = \frac{4\pi \rho V}{c^2} - \frac{\pi f A_s + l_o}{2c} \]  

(A2)

where:
- \( f \) is the analysis band center frequency in Hz.
- \( V \) is volume of the space
- \( A_s \) is the surface area of the space
- \( l_o \) is the edge length of the space
- \( c \) is the speed of sound in air
\[
\beta = \begin{cases} 
1 & \text{simple edge supports} \\
2 & \text{clamped edge supports} \\
\sqrt{2} & \text{realistic mounting conditions}
\end{cases} 
\]

\[
\eta_{12} = \frac{\eta_2}{\eta_1} \eta_{21} 
\]

(A5)

For the flanking path between acoustic volumes 1 and 2, a constant coupling loss factor is assumed.

\[
\eta_{13} = 0.0001 
\]

(A6)

3. Internal Loss Factors
Plate Vibration Damping:

\[
\eta_2 = 2 \zeta 
\]

(A7)

where: \( \zeta \) is damping ratio of the plate

Acoustic Volume Energy Dissipation:

\[
\eta_i = \eta_3 = \frac{2.2}{T_{r_i}} 
\]

(A8)

where: \( T_{r_i} \) is the volume's reverberation time