Identification of Internal Loss Factors During Statistical Energy Analysis of Automotive Vehicles

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ABSTRACT

Statistical Energy Analysis (SEA) is a useful tool for predicting the transmission of noise and vibration through the structures of automotive vehicles. This work discusses the identification of SEA internal loss factor parameters from experimental measurements of vehicle sound pressure levels and structural accelerations. A simple automotive vehicle SEA model can be constructed from elements idealized as uniform beams, flat plates and acoustic volumes. Such an SEA automotive vehicle model can accurately predict the vibro-acoustic response of an automotive vehicles when appropriate equivalent SEA parameters are identified from in situ experimental data. This paper will present an algorithm for identifying internal loss factors for SEA models. The paper will include an example of the application of the algorithm to identification of automotive vehicle internal loss factors from measured vehicle response data. The spectral characteristics of these identified internal loss factors will be compared to those typically employed in Statistical Energy Analysis.

INTRODUCTION

Statistical Energy Analysis (SEA) is becoming a powerful method for quality acoustic design of automobile vehicles for acoustic spaces. SEA predicts vehicle sound pressure and vibration response levels and SEA is most accurate at high frequencies where differential equation based methods often fail. The accuracy of SEA predictions is dependent on the accuracy of SEA model parameters, such as modal density, internal loss factor and coupling loss factor. SEA has a long usage history in aerospace and naval engineering and analytical expressions for SEA parameters are available for regular geometric shapes such as beams, plates, and volumes. Many of the structural details used by naval and aerospace engineers are not applicable to automobiles [Vall, C.F., 1972]. Automobile structures are often made of spot welded sheet metals and the parameters for real assemblies are not accurately approximated by available idealized analytical model subsystems. Currently, measured data is not generally available for automotive vehicle structures. SEA models for automobile applications need to be validated with measured automotive vehicle responses and accurate automotive SEA model parameters determined.

STASTICAL ENERGY ANALYSIS

Statistical Energy Analysis can be illustrated with the simple automotive vehicle idealized as a three element SEA model in Fig. 1. The figure represents vibration and acoustic energy storage by the energies, E1, E2 and E3 in the three SEA element blocks. Vibration and acoustic power flow between the three elements transmitted into the interior through the vehicle structure and flanking paths is shown by the arrows labeled P12, P13, and P23 where the subscripts refer to the elements connected. The external power input P1 comes from engine surface vibration in the engine compartment. Power dissipated by vehicle damping and acoustic treatment is represented by the three arrows Pdiss1, Pdiss2 and Pdiss3.

SEA makes the fundamental assumption that, within narrow frequency bands, the energies in all independent modes equalize at steady state. The total energy in each element, \( E_i \), in a frequency band centered at frequency, \( f \) is derived from the product of the element's modal energy, \( e_i \), and the element's modal density, \( n_i \), at that frequency.
\[ E_i = E_i(f) = n_i(f)\hat{v}_i(f) \]  \hspace{1cm} (1)

The total element energy can be used to compute the expected value of mean-rms velocity for structural elements,

\[ E_i = m_i \left\langle \hat{v}_i^2 \right\rangle \]  \hspace{1cm} (2)

or the expected value of mean-rms sound pressure in acoustic spaces,

\[ E_i = \frac{V}{p c^3} \left\langle \hat{p}^2 \right\rangle \]  \hspace{1cm} (3)

The SEA model equations derived from conservation of power and equal modal energies take the form of simultaneous, frequency-dependent, algebraic equations. For a \( K \) element model, they can be written as:

\[ N \mathbf{e} = \left( \frac{1}{2 \pi f} \right) \mathbf{P} \]  \hspace{1cm} (4)

where: \( \mathbf{e} = [e_1 \ldots e_K]^T \), modal energy vector (Joules/mode)

\[ N = N(\eta_i, \eta_j, \eta_k) \], the SEA coefficient matrix

\[ n_i = n_i(f) = \text{element internal loss factors} \]

\[ n_{ij} = n_{ij}(f) = \text{\( i \)th to \( j \)th coupling loss factor} \]

\[ n_i = n_i(f) = \text{modal density (modes/Hz)} \]

\( f \) = analysis band center frequency (Hz)

\[ \mathbf{P} = [P_1(f) \ldots P_K(f)]^T \], the vector of element input powers (Watts)

The SEA coefficient matrix, \( N = [N_{ij}] = N(\eta_i, \eta_j, \eta_k) \), is a symmetric matrix constructed by SEA parameters modal density \( n_i \), internal loss factor \( \eta_i \), and coupling loss factor \( \eta_{ij} \).

\[
N = \begin{cases} 
  n_i \eta_i + \sum_{k=1}^{K} n_i \eta_{ik} & i = j = 1 \\
  n_i \eta_k + \sum_{k=1}^{K} n_i \eta_{ik} & i = j = K \\
  n_i \eta_i + \sum_{k=1}^{i-1} n_i \eta_{ki} + \sum_{k=i+1}^{K} n_i \eta_{ik} & i = j, \ i \neq K \\
  -n_i \eta_{ii} & \ i \neq j \text{ and } i > j \\
  N_{ji} = N_{ij} & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (5)

**SEA PARAMETER AND IDENTIFICATION**

Total energies are the mean-rms energies in each element. The input power is the external power into the element. The internal loss factors quantify the rates of energy dissipation from each element. The coupling loss factors govern the power flow from one element to another. The SEA analysis problem is to use (4) and (1) to solve for the energy responses of each element given the element SEA parameters and input powers.

The SEA parameter identification problem is an inverse problem which uses the power balance equations and measured values of element response energies and input powers to compute SEA parameter values. The SEA equations are linear in the SEA coefficients \( N_{ij} \). For each test with a single measurable power input, a maximum of \( K \) values in the coefficient matrix \( N \) can be computed. With access to all element input powers, \( K \) independent tests are possible, and \( K^2 \) values in the coefficient matrix can be computed. For a general SEA model with all possible element connections, there are \( K \) internal loss factors, \( K \) modal densities and \( K(K-1)/2 \) independent coupling loss factors. For the simplest practical case of two elements \((K = 2)\), there are five SEA parameters to identify at each band frequency: two internal loss factors, two

---

**Table 1: Number of Independent SEA Tests versus Model Size**

<table>
<thead>
<tr>
<th>Model Size ( K )</th>
<th>Possible Tests ( K^2 )</th>
<th>Loss Factors ( K )</th>
<th>Modal Densities ( K )</th>
<th>Coupling Factors ( K(K-1)/2 )</th>
<th>Total SEA Parameters ( K(K+3)/2 )</th>
<th>Tests - Parameters ( K(K-3)/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
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<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
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<td>6</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>
modal densities and one coupling loss factor. With a two element model there are only four possible tests with which to find these parameters (Table 1). The noteworthy fact here is that independent identification of all SEA parameters requires models with three or more elements for a model with all element couplings present.

Previous investigations of SEA parameter identification have assumed input power is measurable, SEA models are limited to two or three elements and elements have regular geometric topology. Bies and Hamid (1980) conducted an experiment to identify internal loss factors and the coupling loss factor for two coupled plates. Clarkson and Pope (1981) developed an indirect experimental method to determine the modal density and internal loss factors for flat plates and cylinders. Ghering and Raj (1987) conducted an experimental investigation on a cylinder-plate-beam structure. Norton and Greenhalph (1986) identified the internal loss factors of lightly damped SEA model of a pipeline system by both steady-state power flow and burst random-noise techniques.

The work discussed here will discuss parameter verification for more general, and larger, SEA models. For this initial investigation only internal loss factors will be identified and the simplifying assumption will be made that all coupling loss coefficients are known. This assumption is justified for many structures because SEA coupling coefficients are related to structure geometry, mass and stiffness properties. Those values are typically well determined by the structure's design. In contrast, structure energy dissipation models are typically well determined by structure geometry and material properties. Structure energy dissipation models are more typically determined empirically and it is these dissipation models which are needed to determine the SEA internal loss factors. The discussion below will present a method for identifying the $K$ independent SEA internal loss factors of a $K$ element model from a test with one known power input through a single SEA element. Additionally, the measurement of input power is often difficult and the identification of input power from measured element energies will be presented.

INPUT POWER AND INTERNAL LOSS FACTOR IDENTIFICATION

SEA response energies, $E_i$, are measurable quantities in terms of rms vibration velocity and sound pressure level. In contrast input powers, $P_i$, are not easily measurable without using an impedance head and external excitation source. We will assume here that responses from all elements in an SEA model are measurable at each band center frequency of interest, and that all coupling loss factors and modal densities are known at these frequencies. For convenience, we will assume with generality that power is input to element 1. We will now use the SEA model equations solve for input power to element one and internal loss factors for all other elements.

Rewrite the SEA equation (4) to explicitly remove the input element's energy response from the SEA coupling matrix.

\[
\begin{bmatrix}
N_{11} & N_{12} & \cdots & N_{1K} \\
N_{12} & N_{22} & \cdots & N_{2K} \\
\vdots & \vdots & \ddots & \vdots \\
N_{1K} & N_{2K} & \cdots & N_{KK}
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2 \\
e_3 \\
\vdots \\
e_K
\end{bmatrix}
= 
\begin{bmatrix}
P_1 \\
2\eta \sum_{i=1}^{K} N_{1i} e_i
\end{bmatrix}
\]

(6)

Removing the input power element's equation from the SEA model equations removes the input power from the model equations. The power input element's equation is rearranged (6) to give the elements input power in terms of known element energies, coupling losses, and the input element's internal loss factor.

\[
P_1 = 2\eta \sum_{i=1}^{K} N_{1i} e_i
\]

(7)

The internal loss factor for the powered element, $\eta_1$, appears in the right hand side of this equation in $N_{1i}$ and is needed to solve for input power, $P_1$. The internal loss factor for the powered element with unknown input power cannot be identified from measured element energies and we must use other methods to estimate it. The remaining SEA model equations are rearranged in a symmetric form which parallels the original SEA model equation but has the SEA model size reduced by one.

\[
\begin{bmatrix}
N_{12} & \cdots & N_{1K} \\
N_{22} & \cdots & N_{2K} \\
\vdots & \ddots & \vdots \\
N_{K2} & \cdots & N_{KK}
\end{bmatrix}
\begin{bmatrix}
e_2 \\
e_3 \\
\vdots \\
e_K
\end{bmatrix}
= 
\begin{bmatrix}
N_{12} \\
N_{13} \\
\vdots \\
N_{1K}
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2 \\
\vdots \\
e_K
\end{bmatrix}
\]

(8)

The modified coupling matrix, $\hat{N}$, is produced by removing the row and column associated with the power input element. The reduced response energy vector, $\hat{e}$, is produced by removing the power input element's energy. An equivalent power input vector, $\hat{P}$, is then written in terms of the input element's response energy, known coupling factors for that element and the input element's internal loss factor. The above formulation allows an SEA identification problem with unknown input power to be recast into the form (8) with known inputs.

\[
\hat{N}\hat{E} = \hat{P}
\]

(9)

where:

\[
\hat{P} = 
\begin{bmatrix}
N_{12} \\
N_{13} \\
\vdots \\
N_{1K}
\end{bmatrix}
\hat{e}_i = 
\begin{bmatrix}
-N_{12} e_1 \\
-N_{13} e_1 \\
\vdots \\
-N_{1K} e_1
\end{bmatrix}
\]

\[
\hat{e} = 
\begin{bmatrix}
e_1 \\
e_2 \\
\vdots \\
e_K
\end{bmatrix}
\]
in the revised SEA model, the vector $\vec{P}$ is the new SEA model input vector.

To identity values for the internal loss factors, $\eta_i$, from measurements of the element energies, $E_i$, use (5) and solve for $\eta_i$ in $N_{ie}$ via:

$$
\eta_i = \frac{1}{n_i e_i} \left\{ \sum_{k=1}^{i-1} n_k \eta_{ik} (e_k - e_i) \right\} + \sum_{k=i+1}^{K} n_i \eta_{ik} (e_k - e_i) + \sum_{j=i+1}^{K} n_j \eta_{kj} (e_j - e_i)
$$

where: $e_i = E_i/n_i$

### APPLICATION EXAMPLES

The first example is a simple conceptual model (Fig. 2) to illustrate the details of internal loss factor identification. The SEA model consists of two identical cubes, separated by an aluminum panel with a flanking path between the cubes to model energy transfer by paths other than through the panel. Acoustic powers are input to the space 1 and the working medium is air (Table 2). Space #1 is designated as element 1, the panel as element 2 and space #2 as element 3.

Computation of the SEA model parameters, such as modal densities and coupling loss factors for every element and coupling is the first step in internal loss factor identification. The modal density [Appendix] for structural element 2

$$
n_2 = \frac{\sqrt{3}A}{hc_i} = 0.007 \times 5182
$$

$$
= 11.9 \times 10^{-3} \text{ (modes/Hz)}
$$

The modal densities at 5,000 Hz [Appendix] for acoustic elements 1 and 3 are

| Table 2: Example 1 Geometry, Material Property, Assumed Measurement at 5,000 Hz. |
|----------------------------------|----------|
| Longitudinal Wave Speed in Panel (m/s) | 5182. |
| Speed of Sound in Air (m/s) | 344. |
| Panel Thickness (m) | 7.00E-03 |
| Length of the Cube (m) | 5.00E-01 |
| Air Density (kg/m³) | 1.244 |
| Aluminum Density (kg/m³) | 2700. |
| Pressure in Space #1 @ 5000Hz (N/m²) | 39. |
| Velocity in Panel @ 5000Hz (m/s) | 8.85E-04 |
| Pressure in Space #2 @ 5000Hz (N/m²) | 19.41 |

$$
n_1 = n_3 = \frac{4\pi f^2 V}{c^3} + \frac{\pi f^2 A_0}{2c^2} + \frac{L_k}{8c} = \frac{4\pi \times 5000^2 \times 0.5^3}{344^3} + \frac{\pi \times 5000 \times 1.5}{344^2} + \frac{6}{8 \times 344}
$$

$$
= 1.10 \text{ (modes/Hz)}
$$

The critical frequency is 1786 Hz and the band frequency is 5,000 Hz. The coupling loss factors [Appendix] from the structural element 2 to the acoustic element 3 are

$$
\eta_{21} = \eta_{33} = \frac{\rho_{\text{air}} c_{\text{air}} A}{M_0 \omega \left(1 - \frac{f}{f_c}\right)^{1/2}}
$$

$$
= \frac{1.244 \times 344 \times 0.25}{2700 \times 0.007 \times 0.25 \times 2 \pi \times 5000 \sqrt{1 - \frac{1786}{5000}}}
$$

$$
= 9.0 \times 10^{-4}
$$

The coupling loss factor from acoustic element 1 to the structural element 2 can be computed by the reciprocity relationship [Lyon, R.H., 1975].

$$
\eta_{12} = \frac{n_2}{n_1} \eta_{21} = \frac{11.94 \times 10^{-3}}{1.1} 9.0 \times 10^{-4}
$$

$$
= 1.0 \times 10^{-5}
$$

The coupling loss factor for the flanking path between element 1 and element 3 is assumed constant for this example.

$$
\eta_{13} = 0.0001
$$

The internal loss factor identification is carried out by using (10) to compute the internal loss factors. For this example assume the measured energy values at 5,000 Hz.

![Figure 2: SEA model Schematic for Example 1](image-url)
\[
E_i = \frac{V}{\rho c^2} \left( \langle p^2 \rangle \right) = \frac{0.125}{1.244 \cdot 344^2} \left( 39.13 \right) \nonumber
= 1.3 \times 10^{-3} \text{ (Joules/Hz.)} \tag{16a}
\]

\[
E_2 = m \langle v^2 \rangle \nonumber
= 2700 \cdot 0.0075 \cdot 0.5 \left( 8.85 \times 10^{-4} \right)^2 \nonumber
= 3.7 \times 10^{-6} \text{ (Joules/Hz.)} \tag{16b}
\]

\[
E_3 = \frac{V}{\rho c^2} \left( \langle p^2 \rangle \right) = \frac{0.125}{1.244 \cdot 344^2} \left( 19.41 \right) \nonumber
= 3.2 \times 10^{-4} \text{ (Joules/Hz.)} \tag{16c}
\]

The element modal energies, \( e_i \), are computed by dividing measured total energy, \( E_i \), by the modal density, \( n_i \).

\[
e_1 = \frac{E_1}{n_1} = \frac{1.3 \times 10^{-3}}{1.1} \nonumber
= 1.2 \times 10^{-3} \text{ (Joules/mode)} \tag{17a}
\]

\[
e_2 = \frac{E_2}{n_2} = \frac{4.0 \times 10^{-6}}{11.94 \times 10^{-3}} \nonumber
= 3.1 \times 10^{-4} \text{ (Joules/mode)} \tag{17b}
\]

\[
e_3 = \frac{E_3}{n_3} = \frac{3.2 \times 10^{-4}}{1.1} \nonumber
= 3.0 \times 10^{-4} \text{ (Joules/mode)} \tag{17c}
\]

Equations (11) through (17) are substituted into (10) to yield the values of internal loss factors, \( \eta_2 \) and \( \eta_3 \).

\[
\eta_2 = 8.1 \times 10^{-5}, \tag{18a}
\]

\[
\eta_3 = 3.1 \times 10^{-4}. \tag{18b}
\]

The internal loss factor in element 1 cannot be independently computed because we have used measured energy in that element as our model input.

The second example is an automotive vehicle to further demonstrate the internal loss factor identification algorithm. The vehicle is modeled by 11 SEA elements and 15 couplings (Fig. 3). Structural vibration energy storage models the "Engine" block, left rail frame (LRail), right rail frame (RRail), "Hood", Front of "Dash", "Floorpan", and "Body" panels. Acoustic pressure energy storage models the engine compartment (EngComp), under car volume (UndCarVol), over car volume (OvrCarVol), and interior (Interior). The model input power is applied to the "Engine" block only but was not measured in this example and the resulting measured "Engine" vibration energy is used as the model input.

The laboratory test condition used was vehicle operation under load at 3000 rpm on a dynamometer. The RMS sound pressure levels in acoustic volumes and RMS acceleration levels in structural elements are measured in 4-12 spatial positions corresponding to each element in the model and the measured RMS velocity and pressure responses are used to identify the internal loss factors. The identified internal loss factors for under car volume and front of dash are presented in Fig. 4 and 5 respectively.

Constant internal loss factors for structural elements are typical modeling assumptions for most SEA models. Structural internal loss factor corresponds to twice the linear viscous damping ratio for these structures [Lyon, 1975]. Identified internal loss factor values for our automotive structural element example show that the internal loss factors are not a constant at all frequencies. Specifically, the average internal loss factor for the front of dash (FOD) is 0.013 with a large standard deviation of 0.006 (Fig. 4). The average internal loss factor for the FOD results corresponds to a damping ratio of 0.6% which is very small. There is no apparent trend in the identified internal loss factors for the FOD element and the other elements in our model displayed no regular trend as well. This observation indicates that the actual internal loss factor values for automotive structural elements can not be accurately modeled either by Coulomb or viscous friction models. The very small damping measured may partially explain the inability to detect data trends. These results indicate that substantially more experience with identification of automotive structural dissipation is required before these sources of internal energy loss can be accurately modeled.
The internal loss factor for an acoustic element is often modeled using reverberation time.

\[ \eta_1 = \eta_3 = \frac{2.2}{T_s \cdot f} \]  

(19)

where \( T_s \) is the space’s reverberation time

The identified internal loss factor values for the under car volume (UNDCCARVOL) are presented in Fig. 5. A least-squared-error fit to a reverberation time from the identified internal loss factors can be computed.

\[ T_s = \sum_{i=1}^{M} \frac{1}{\eta_i(f_i) / f_i} = 0.27 \text{ sec} \]  

(20)

where \( M \) is the number of frequency bands

\( \eta_i(f_i) \) is the identified internal loss factor at frequency \( f_i \).

The internal loss factors computed (solid line) from the identified reverberation time are also presented in Fig. 5.

Figure 4: Identified and Average Internal Factors for Front of Dash (FOD) Structure

Figure 5: Identified and Computed Internal Loss Factors Using Computed Reverberation Time for Under Car Volume
The acoustic internal loss factors identified for the under car volume (UND CAR VOL) for under car volume are very different from the reverberation time model at lower frequencies: 500 Hz, and 630 Hz. At higher frequencies: 800 to 5,000 Hz, identified internal loss factors closely match the trend corresponding to the values for a reverberation time of 0.27 seconds. This result at high frequencies was true in general for other model acoustic volumes with the measured automotive data. These results indicate that a reverberation time based model of automotive vehicle acoustic spaces is appropriate at higher frequencies where modal density is larger. Our automotive data appeared to follow fixed reverberation times when modal densities were above approximately 0.5 modes per Hertz. Again, additional automotive data are required before modeling trends can be accurately determined.

**SUMMARY**

The SEA internal loss factor identification problem for automotive vehicle model applications is addressed and an algorithm for identifying the internal loss factors is developed. The algorithm is demonstrated by both simple illustrative example and more complicated automotive vehicle SEA models. Preliminary results show that the common assumption of constant internal loss factors for structural elements may not accurately represent actual system responses, however, reverberation time models for acoustic volume elements are probably justified at higher frequencies. Additional automotive measurement experience is required before more accurate modeling trends can be determined.

**ACKNOWLEDGMENT**

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**REFERENCES**


**APPENDIX**

**Modal Density Equations** (Lyon, R.H., 1975):

**Plate Structure:**

\[
\eta_2 = \frac{\sqrt{3}A}{hc_l}
\]

(A1)

where 
- \(A\) is the area of the panel (m^2)
- \(h\) is the thickness of the panel (m)
- \(c_l\) is longitudinal wave speed in the panel (m/s)

**Acoustic Space:**

\[
\eta_1 = \eta_3 = \frac{4\pi^2 V}{c^3} + \frac{\pi A_{e}}{2c^2} + \frac{L_e}{8c}
\]

(A2)

where 
- \(f\) is the analysis band center frequency (Hz).
- \(V\) is volume of the space (m^3)
- \(A_e\) is the surface area of the space (m^2)
- \(L_e\) is the edge length of the space (m)
- \(c\) is the speed of sound in air (m/s)

**Coupling Loss Factor Equations** (Lyon, R.H., 1975):

From Plate to Acoustic Space:
\[ \eta_{21} = \eta_{23} = \frac{\rho c A}{2\pi f M_p} \left( \frac{2\lambda_c^2 P}{\pi^2 A} \sin^{-1} \left( \frac{f}{f_c} \right) \beta, \quad f < f_c \right) \]

\[ \left( \frac{f_c}{f} \right)^{-1/2}, \quad f > f_c \]

where \( \rho \) is fluid density (kg/m\(^3\)),

\( c \) is the speed of sound in the fluid (m/s),

\( A \) is the area of the panel (m\(^2\)),

\( P \) is the perimeter of the panel (m),

\( M_p \) is the mass of the panel (kg)

\( f_c = \frac{12.5}{h} \) the critical frequency (Hz.)

\( \lambda_c = \frac{c}{f_c} \), the wavelength of sound at the critical frequency (m)

\( \beta = \begin{cases} 1 & \text{for simple edge supports} \\ 2 & \text{for clamped edge supports} \\ \sqrt{2} & \text{for typical mounting conditions} \end{cases} \)