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DEVELOPMENT OF A COMPARISON INDEX AND A DATABASE FOR SEA MODEL RESULTS

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ABSTRACT

This study analyzes methods of comparing SEA model results with experimental results for key traits. These qualitative traits provide the basis for correlation of model results with experimental results through the development of a comparison index. This paper formulates a comparison index and illustrates the application to SEA models. A customized data structure was designed around the comparison index to store all necessary aspects of the modeling, experiment and comparison results. This data structure was then implemented using relational database software. These new tools; the comparison index and the SEA database, will create a common language and a forum for SEA model results that will aid and stimulate dialog in the SEA modeling community and in tern, advance the science of SEA modeling.

INTRODUCTION

Current trends in advanced vibro-acoustic analytical design are moving towards analytically based Statistical Energy Analysis (SEA) modeling early in the prototyping phase of design. To create successful SEA models without experimental results, a modeler must rely on experience and previous model results. This creates a need for a system that characterizes previous model results and organizes them in a structured format.

Developing theoretically accurate SEA models, and verifying them through experimental testing is the emphasis at the System Dynamics Laboratory at Michigan State University. Software developed in this program uses experimental measurements to determine optimum loss factors to ensure agreement between experimental and model results. This process produces large volumes of numerical and experimental data that must be carefully analyzed to determine the correlation between the theory and experiment. Over the years, the acquisition of a wide range of experimental and model data has identified the need for a system of quantitatively comparing and storing results for reference. Two needs have been identified. First, a method is needed for characterizing the correlation between experimental and model results. Second, a customized database is needed for cataloging results. This paper discusses the development of these tools.

Traditional methods for comparing SEA model results with experimental results consist of graphically comparing the data. Then a qualitative judgment is made based on this comparison. However there is no convention on how the comparison is performed and a graphical representation of the results is usually the final result. If a conventional set of numerical parameters were proposed that characterized the important traits of the comparison then an index could be used in addition to or in place of the graphical results. Furthermore, such a tool would provide SEA modelers with a vocabulary for discussing SEA model and experimental results. The first result of this study is a Comparison Index for SEA model and experimental results comparisons.

SEA modeling and experimental testing generates large quantities of numerical results that must be carefully organized in order to be used effectively. For example, results can be useful when building new SEA models. Elements from previous models which have proven successful can be retrieved from the records and applied to new models. In this way, a library of SEA elements can be assembled, providing a valuable resource for future modeling efforts. However, due to the quantity of data associated with SEA models, careful consideration must be used when designing a data archiving system. The second result of this study is the structure of a customized database for archiving SEA model and experimental results. While existing software has been developed using database structures [Park, 1987], this database focuses on using the comparison index as a central component in element and model verification.

SEA MODEL VERIFICATION

A Statistical Energy Analysis (SEA) model is verified by comparing the results predicted by the SEA model with the measured results from experimental testing. A typical SEA model, composed of many substructures, predicts the structural velocity and acoustic pressure levels (response) of a structure from a description of the system's physical parameters and excitation. The SEA results can then be compared to the measured results for each element in the model.

SEA model and measured response results are traditionally compared with a graph showing both sets of data for each element. The response levels are plotted against the third octave band center frequencies indicating the level of agreement over the frequency spectrum. For example, Figure 1 is a typical comparison of an SEA model element and experimental data. The graph shows that the SEA model results match the experimental results very well for low frequencies (below 1600 Hz), but there is some disagreement above 2000 Hz. An SEA modeler may conclude from these observations that the model should be modified so that there is better agreement above 2000 Hz. Conversely, the modeler may conclude that these results are satisfactory depending on the needs of the data. In either case, the conclusion is based on qualitative judgment of the data represented on the graph.

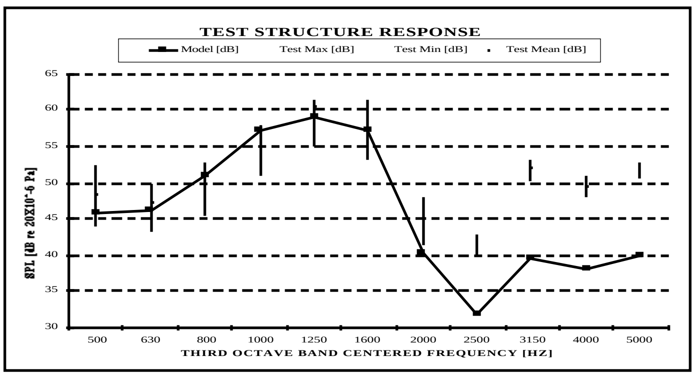

Figure 1. Typical Graphical Comparison of Experimental Data and SEA Model Results

COMPARISON INDEX

A comparison index has been developed to remove the subjectivness from the evaluation. If the data is compared to the comparison index and the results fail to meet the specification, then conclusions will depend less on subjective judgment. A numerical index also allows the designer to keep a record of this information to learn from experience and to aid in development of new models.

QUALITIES OF A USEFUL ELEMENT COMPARISON INDEX - The key properties of a useful Comparison Index include: mean deviation, upper and lower bound and normalization. The mean deviation is a measure of the difference between the experimental and model results. It accounts for differences between the SEA and measured results by determining the absolute difference at each center frequency band and summing the error. Normalization refers to the fact that SEA results will vary depending on the element response levels. For example one element may have large response levels in a certain frequency band and a different element may have low response levels in the same band. While the two cases have different quantitative responses, one can still ask which model is more accurate. If the results are not normalized to account for the difference in response levels, then the comparison may be biased by the level of response of an element. The upper and lower bounds refer to a description of the maximum and minimum value of the response levels of the results. This information indicates the range of response levels, and is necessary to distinguish between cases in which the mean deviation is similar for two models, but the measured response levels of one model are significantly larger. In such a case the mean deviation would be different relative to the range of response levels. These concepts are incorporated into the numerical element comparison index below.

A NUMERICAL COMPARISON INDEX - The comparison index proposed in this paper is a function of 3 parameters that indicate how well the SEA model element results agree with the experimental data. The first two parameters describe the spread of measured data for each element. Although they deal with qualitative measures of the element response, they are needed as a reference for the mean deviation parameter. They give the range in the form of a maximum and minimum extreme values of the measured response levels. The third parameter is the normalized mean deviation of the element response. The comparison index gives the mean deviation relative to the range of the data and indicates how well the SEA data agrees with the measured data independent of the response levels of the element. These parameters, named α_j, β_j, and γ_j along with the element comparison index δ_i and model comparison index, Δ will be derived below.

The measured mean squared (MS) response is the basis for all model verification studies [Lyons, 1975]. The objective of the proposed comparison index is to compare the SEA and measured response for all frequencies with a small set of parameters. The experimental acoustic measurement data consists of the measured response at many different locations for each element, on the test structure. These measurements are represented by $R_j(i,k)$, where R is the j^{th} element MS response at the i^{th} location for the k^{th} frequency band. The MS measured response for the k^{th} third octave band,

$$e_i^m(f_k) = \frac{1}{N_m}\sum_{k=1}^{N_m} R_i^2(k, f_k) \tag{1}$$

where N_m represents the number of measurement locations for element i.

By summing over each third octave band, the mean measured MS response,

$$e_{i\ mean}^{m} = \frac{1}{N_f} \sum_{f_k=1}^{N_f} e_i^m(f_k) \tag{2}$$

where N_f is the number of frequency bands measured.

The maximum normalized MS response, α_j of the experimental data is the first parameter needed. The largest $e_i^m(f_k)$ in the measured data set normalized by the mean measured MS response,

$$\alpha_i = \frac{Max_{f_k}\left(e_i^m(f_k)\right)}{e_{i\ mean}^{m}} \tag{3}$$

Similarly, the minimum normalized MS response, β_i is given by the smallest $e_i^m(f_k)$ in the measured data set normalized by the mean measured MS response,

$$\beta_i = \frac{Min_{f_k}\left(e_i^m(f_k)\right)}{e_{i\ mean}^{m}} \tag{4}$$

These parameters are normalized by the mean measured MS response in order to make them unitless and to make the numerical scale of the parameters a convenient size.

The mean deviation, γ_i is computed by taking the mean square of the difference between the SEA and the measured response levels, normalized by the measured response level for each center frequency band in linear scale, summing the error over all frequency bands, taking the square root and dividing by the number of bands,

$$\gamma_i = \frac{1}{N_f}\sqrt{\sum_{k=1}^{N_f}\left[\frac{e_i^m(f_k) - e_i^p(f_k)}{e_i^m(f_k)}\right]^2} \tag{5}$$

where e_i^p represents the predicted SEA response. The normalization is important for the parameter γ_i. The reason is that the energies in the elements near power input sources are larger than the energies in the elements which are far from the power input sources. The normalization weighs the elements with smaller energy response equally in the evaluation. Otherwise, the difference will be dominated by the energies in a few elements near sources. The important elements often have the lowest element energies because they are the elements at the end of the power flow chain.

Finally, the element comparison index, is computed by taking the ratio of the mean deviation to the difference between the maximum and minimum normalized MS responses,

$$\delta_i = \frac{\gamma_i}{\alpha_i - \beta_i} \tag{6}$$

A more successful element will have a small δ_i.

The mean of the element indices for an entire model can be used as the Model Comparison Index,

$$\Delta = \frac{1}{j}\sum_{i=1}^{j}\delta_i \tag{7}$$

where j is the number of elements in the model. A successful model will minimize the error between the SEA response and measured response of each element, represented by δ_i. It will also spread the error equally between elements. This property can be determined by comparing Δ for the model and δ_i for each element. A more successful model will have a small Δ.

This concludes the derivation of the Element and Model Comparison Indices. A numerical example is given below to illustrate the application of these equations.

APPLICATION OF THE COMPARISON INDEX

Three examples are given below, that illustrate how the Element and Model Comparison Index are applied. The examples illustrate 2 important features of the Comparison Index. First, elements with smaller mean deviation between the SEA response and measured response levels give smaller values for δ_i, which indicates a more successful model. Second, elements with similar mean deviation but a larger range of measured response levels over the third octave center bands give smaller values for δ_i. The third example illustrates the use of the Model Comparison Index.

The first example illustrates that the Element Comparison Index is smaller for models with less mean deviation. The examples consist of simplified sets of data with only 3 center frequency bands. This is sufficient to illustrate the conceptual aspects of the Comparison Index. In Figure 2, models of 2 elements are compared. Model A has a small deviation between the model and measured response levels in each frequency band and Model B has a larger deviation in each frequency band. Clearly, Model A is superior to Model B because the deviation between the model and the measured response levels are smaller for each frequency band. This result is indicated by the Comparison Index results given in Tables 1 and 2, which shows that the Element Comparison Index is smaller for Model A then for Model B.

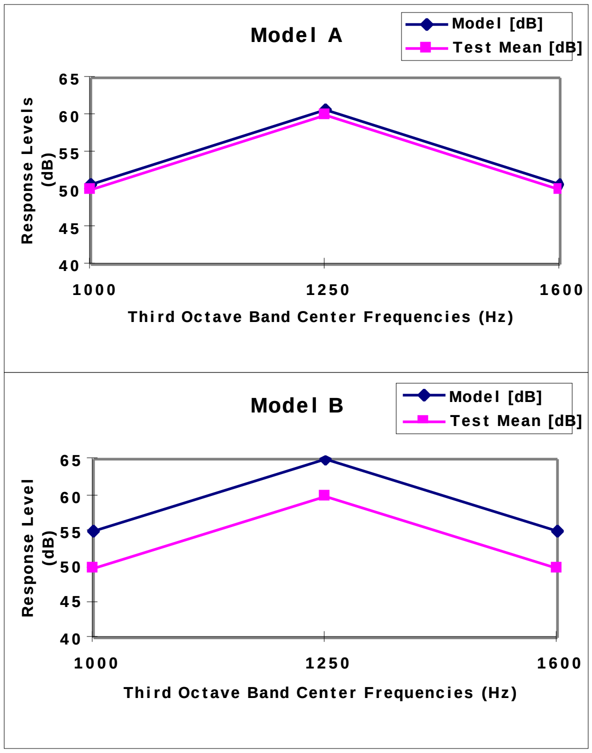

Figure 2. Case 1, Model A is superior to Model B because it has less deviation.

Table 1. Case 1 Model A

Band [Hz]	Measured MS Response	Predicted MS Response	Error
1000	100E+3	126E+3	0.067
1250	1.00E+6	1.26E+6	0.067
1600	100E+3	126E+3	0.067
Mean Measured Response Level			400E+3
Maximum Normalized MS Response			2.50
Minimum Normalized MS Response			0.25
Mean Deviation			0.15
Element Comparison Index			0.066

Table 2. Case 1 Model B

Band [Hz]	Measured MS Response	Predicted MS Response	Error
1000	100E+3	316E+3	4.68
1250	1.00E+6	3.16E+6	4.68
1600	100E+3	316E+3	4.68
Mean Measured Response Level			400E+3
Maximum Normalized MS Response			2.50
Minimum Normalized MS Response			0.25
Mean Deviation			1.25
Element Comparison Index			0.56

The second example illustrates the effect of the range of measured response levels on the Element Comparison Index. In Figure 3, models of 2 different elements are compared to determine which model more accurately predicts the measured response. In this case, Model A and B have the same deviation between the model and the measured response levels for each frequency band. Model A, however, has a significantly larger range of response levels than Model B. In this case it is logical to conclude that Model A is superior to Model B because the deviation, relative to the span of the data is smaller. This becomes more obvious when Model B is shown with the graph spanning the full range of response levels. This result is indicated by the Comparison Index results given in Tables 3 and 4, which shows that the Element Validation Index is smaller for Model A than for Model B.

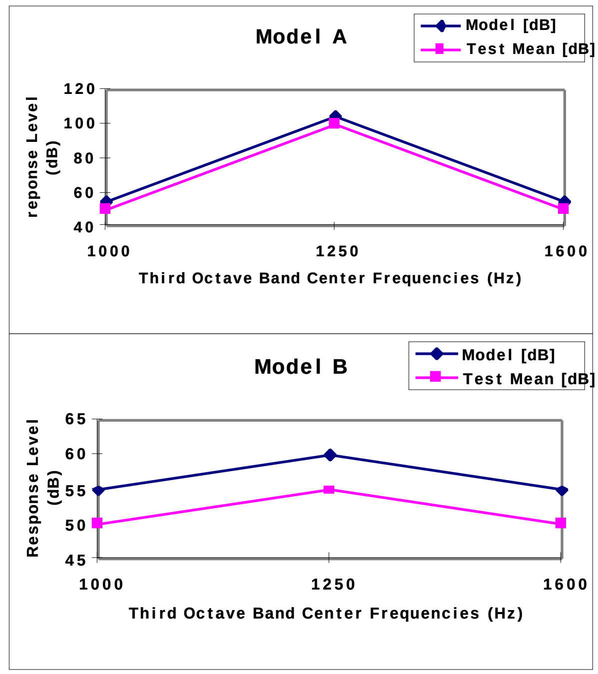

Figure 3. Model A is superior to Model B because the deviation is smaller relative to the span of the data.

Table 3. Case 2 Model A

Band [Hz]	Measured MS Response	Predicted MS Response	Error
1000	100E+3	316E+3	4.68
1250	10.0E+9	31.6E+9	4.68
1600	100E+3	316E+3	4.68
Mean Measured Response Level			3E+9
Maximum Normalized MS Response			3.00
Minimum Normalized MS Response			0.00
Mean Deviation			1.25
Element Comparison Index			0.42

Table 4. Case 2 Model B

Band [Hz]	Measured MS Response	Predicted MS Response	Error
1000	100E+3	316E+3	4.68
1250	316E+3	1.00E+6	4.68
1600	100E+3	316E+3	4.68
Mean Measured Response Level			172E+3
Maximum Normalized MS Response			1.84
Minimum Normalized MS Response			0.58
Mean Deviation			1.25
Element Comparison Index			0.99

The third example shows the computation of the Model Comparison Index. The Element and Model Comparison Index results for 2 different 8 element models are given in Table 5.

Table 5. Model Comparison Index Results for 2 Different 8 Element Models

Element Number	Model A Element Comparison Index	Model B Element Comparison Index
1	0.86	0.98
2	0.00	0.00
3	1.12	2.31
4	0.84	1.21
5	0.41	0.65
6	0.41	0.65
7	0.41	0.40
8	0.41	0.21
Model Comparison Index	**0.56**	**0.80**

Clearly Model A is superior to Model B because the Element Comparison Index for most of the elements is smaller than for Model B. This is indicated by the Model Comparison Index. This data also highlights the fact that there is significantly more error in the elements 3 and 4 in Model B than in the other elements in Model B, which contribute to the larger Model Comparison Index. The Element Comparison Index data shows that the error is shared more equally by all elements in Model A.

DEVELOPMENT OF SEA MODEL DATABASE

SEA modeling and experimental testing generates large quantities of numerical results that must be organized in a structured manner to be of use in future modeling efforts. One important use for past SEA data is in the creation of new SEA models. When an existing model is being developed, a physical system often exists for comparison. The model results can be compared to the experimental results and the model can be incrementally improved until satisfactory results are obtained. This is not the case when a model is being developed early in the design process, before a physical prototype has been created. When the structure has not yet been fabricated a modeler must rely on experience to choose parameters for elements in the model. This process would be easier if the modeler had a well organized catalog of SEA elements from previous models. The modeler could choose from a list of elements based on the physical parameters and the success of the element in previous models.

An SEA model database should include both the physical parameters that define the element, the comparison index of each element which indicates how successful the element performed in previous models, and information about the model which used the element, including the Model Comparison Index. This format suggests the use of a relational database with 2 data structures: element and model data structures.

Element data consists of the physical parameters that define the element. For example a plate elements would include: thickness, surface area, mass, etc... Also included in the element data is the Comparison Index data. Model data consists of the identification of each element in the model and the Model Comparison Index. Element connectivity data may also be included in the model data.

A relational database can take advantage of such a data structure because it can accommodate multiple data types and use the structure in several ways. If one element is used in many models, it need only be defined once. This results in a compact data structure. It also allows for flexible search options. A modeler can look up the names of all models that include a particular element. Also, since the Model Comparison Index computation consists of data that is contained in the element data, it can be automated.

Ultimately, the SEA database can provide a valuable platform for modeling. It could be integrated into SEA modeling software so that the elements can be drawn from the database and automatically inserted into the model. It could provide a forum for presenting and comparing SEA model results through the World Wide Web. Modelers with access could post their results to a common database that could be viewed by and borrowed from by other modelers. The possibilities are endless. However, to take advantage of these concepts, the SEA community must begin by adopting a set of common conventions, such as the comparison index, and element and model data structures presented here.

DATABASE IMPLEMENTATION - Relational database software was used to implement the element and model data structures. Figure 4 shows the Element Data Record for a typical plate element. It includes data fields for a unique element name and description, the element type (plate, beam, acoustic volume, etc.), the model in which the element is included, the Comparison Index data, and the physical parameters that define the element. Figure 5 shows the Model Data Record for a typical model. It includes a data field for a model name and description, the Model Comparison Index data and a list of the elements included in the model.

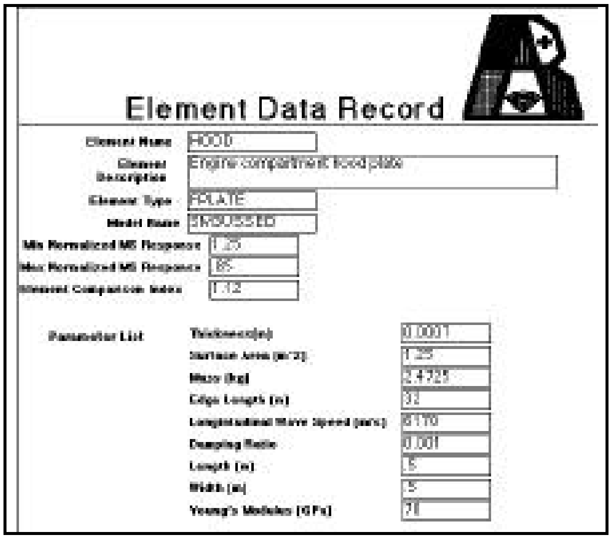

Figure 4. Element Data Record.

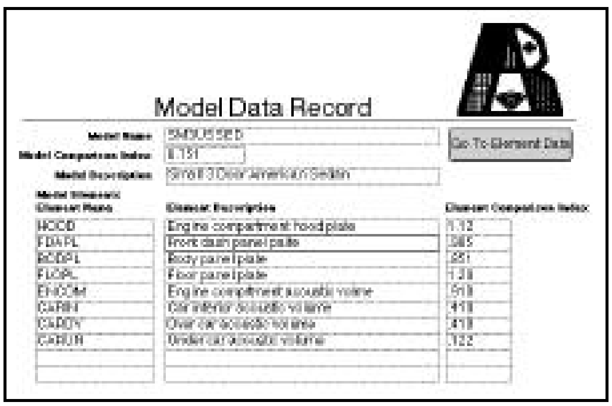

Figure 5. Model Data Record.

The relational database structure has features that can simplify the data archiving process. For example, the Model Comparison Index can be automatically computed from the Element Comparison Index data. This customized database will help to organize and document SEA results and lead to more rapid development in the early stages of design.

CONCLUSION

The comparison index and SEA database structure developed in this paper are useful tools for SEA modeling. The comparison index allows the model designer to report the success of individual elements and the entire model through a small number of indices and provides a method for quantitative comparison of models. It provides a measure of the success of previous model elements that can be used when developing new models. The customized database including multiple data structures provides a well structured platform for archiving and retrieving SEA data that can also aid in the development of new SEA models early in the design process. These new tools will create a common language and a forum for SEA model results that will aid and stimulate dialog in the SEA modeling community and in tern, advance the science of SEA modeling.

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