C:\Users\beck\Desktop\DellDesktop\exact\1D Cartesian\ XI0B1T0   , I = 1, 2, 3
James V. Beck, Feb. 23, 2012, Mar. 21, 2012

Several semi-infinite solutions are considered in these notes. The distance L is a nominal distance since no character distance is present in the problem. (Actually $\sqrt{\alpha t}$ has the dimensions of meter, but we chose instead to use L to be compatible with the finite body cases.)

X10B1T0, X10B0T1 and X10B1T1

For a step change in the surface temperature at the surface of a semi-infinite body, the dimensional mathematical description of the **X10B1T1** is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \quad 0 < x < \infty,\ t > 0 \qquad \text{(X10-1)}$$

$$T(0,t) = T_w \qquad \text{(X10-2)}$$

$$T(\infty,t) \text{ is finite} \qquad \text{(X10-3)}$$

$$T(x,0) = T_{in} \qquad \text{(X10-4)}$$

Dimensionless groups for the **X10B1T0** problem

$$\tilde{T}_{X10B1T0}(\tilde{x},\tilde{t}) = \frac{T(x,t)-T_{in}}{T_w - T_{in}},\ \tilde{q} = \frac{q(x,t)L}{k(T_0 - T_{in})},\ \tilde{x} = \frac{x}{L},\ \ \tilde{t} = \frac{\alpha t}{L^2} \qquad \text{(X10-5)}$$

Dimensionless mathematical description

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} = \frac{\partial \tilde{T}}{\partial \tilde{t}}, \quad 0 < \tilde{x} < 1,\ \tilde{t} > 0 \qquad \text{(X10-6)}$$

$$\tilde{T}(0,\tilde{t}) = 1 \qquad \text{(X10-7)}$$

$$\tilde{T}(\infty,\tilde{t}) \text{ is finite} \qquad \text{(X10-8)}$$

$$\tilde{T}(\tilde{x},0) = 0 \qquad \text{(X10-9)}$$

The dimensionless temperature and heat flux solutions are

$$\tilde{T}_{\mathrm{X10B1T0}}(\tilde{x},\tilde{t}) = \operatorname{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right) \qquad \text{(X10-10a)}$$

$$\tilde{q}_{\mathrm{X30B1T0}}(\tilde{x},\tilde{t}) = \frac{1}{\sqrt{\tilde{t}}}\mathrm{e}^{-\frac{\tilde{x}^2}{4\tilde{t}}} \qquad \text{(X10-10b)}$$

For the **X10B0T1** problem dimensionless groups

$$\tilde{T}_{X10B0T1}(\tilde{x},\tilde{t}) = \frac{T(x,t)-T_w}{T_{in} - T_w},\ \tilde{q} = \frac{q(x,t)L}{k(T_{in} - T_w)},\ \tilde{x} = \frac{x}{L},\ \ \tilde{t} = \frac{\alpha t}{L^2} \qquad \text{(X10-11)}$$

Dimensionless mathematical description

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} = \frac{\partial \tilde{T}}{\partial \tilde{t}}, \quad 0 < \tilde{x} < 1,\ \tilde{t} > 0 \qquad \text{(X10-12)}$$

$$\tilde{T}(0,\tilde{t}) = 0 \qquad \text{(X10-13)}$$

$$\tilde{T}(\infty,\tilde{t}) \text{ is finite} \qquad \text{(X10-14)}$$

$$\tilde{T}(\tilde{x},0) = 1 \qquad \text{(X10-15)}$$

The dimensionless temperature and heat flux solutions are

$$\tilde{T}_{X10B0T1}(\tilde{x},\tilde{t}) = \mathrm{erf}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right) \quad \text{(X10-16a)}$$

$$\tilde{q}_{X10B0T1}(\tilde{x},\tilde{t}) = -\frac{1}{\sqrt{\tilde{t}}}\mathrm{e}^{-\frac{\tilde{x}^2}{4\tilde{t}}} \quad \text{(X10-16b)}$$

Note that

$$\tilde{T}_{\mathrm{X10B1T0}}(\tilde{x},\tilde{t}) + \tilde{T}_{\mathrm{X10B0T1}}(\tilde{x},\tilde{t}) = 1 \quad \text{(X30-17)}$$

X20B1T0, X20B0T1 and X20B1T1

The dimensional mathematical description of the **X20B1T1** problem (which has a step change in the surface heat flux) is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \quad 0 < x < \infty,\ t > 0 \quad \text{(X20-1)}$$

$$-k\frac{\partial T}{\partial x}(0,t) = q_0 \quad \text{(X20-2)}$$

$T(\infty,t)$ is finite (X20-3)

$$T(x,0) = T_{in} \quad \text{(X20-4)}$$

Dimensionless groups to reduce to the **X20B1T0** problem

$$\tilde{T}_{X20B1T0}(\tilde{x},\tilde{t}) = \frac{T(x,t)-T_{in}}{q_0 L/k},\ \tilde{q} = \frac{q(x,t)}{q_0},\ \tilde{x} = \frac{x}{L},\ \tilde{t} = \frac{\alpha t}{L^2} \quad \text{(X20-5)}$$

Dimensionless mathematical description of the **X20B1T0** problem

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} = \frac{\partial \tilde{T}}{\partial \tilde{t}}, \quad 0 < \tilde{x} < \infty,\ \tilde{t} > 0 \quad \text{(X20-6)}$$

$$-\frac{\partial \tilde{T}}{\partial \tilde{x}}(0,\tilde{t}) = 1 \quad \text{(X20-7)}$$

$\tilde{T}(\infty,\tilde{t})$ is finite (X20-8)

$$\tilde{T}(\tilde{x},0) = 0 \quad \text{(X20-9)}$$

The temperature and heat flux solutions are

$$\tilde{T}_{X20B1T0}(\tilde{x},\tilde{t}) = \sqrt{4\tilde{t}}\,\mathrm{ierfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right) \quad \text{(X20-10a)}$$

$$\tilde{q}_{X20B1T0}(\tilde{x},\tilde{t}) = \mathrm{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right) \quad \text{(X20-10b)}$$

Dimensionless groups for the **X20B0T1** problem

$$\tilde{T}_{X20B0T1}(\tilde{x},\tilde{t}) = \frac{T(x,t)}{T_{in}},\ \tilde{x} = \frac{x}{L},\ \tilde{t} = \frac{\alpha t}{L^2}$$

Dimensionless mathematical description of the **X20B0T1** problem

$$\frac{\partial^2\tilde{T}}{\partial\tilde{x}^2}=\frac{\partial\tilde{T}}{\partial\tilde{t}},\quad 0<\tilde{x}<\infty,\ \tilde{t}>0 \qquad \text{(X20-6)}$$

$$-\frac{\partial\tilde{T}}{\partial\tilde{x}}(0,\tilde{t})=0 \qquad \text{(X20-7)}$$

$$\tilde{T}(\infty,\tilde{t}) \text{ is finite} \qquad \text{(X20-8)}$$

$$\tilde{T}(\tilde{x},0)=1 \qquad \text{(X20-9)}$$

The temperature and heat flux solutions are

$$\tilde{T}(\tilde{x},\tilde{t})=1 \qquad \text{(X20-10a)}$$

$$\tilde{q}(\tilde{x},\tilde{t})=0 \qquad \text{(X20-10b)}$$

X30B1T0, X30B0T1 and X30B1T1

For a step change in the ambient temperature at the surface of a semi-infinite body, the mathematical description (**X30B1T1**) is

$$\frac{\partial^2 T}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t},\quad 0<x<\infty,\ t>0 \qquad \text{(X30-1)}$$

$$-k\frac{\partial T}{\partial x}(0,t)=h_1(T_f-T(0,t)) \qquad \text{(X30-2)}$$

$$T(\infty,t) \text{ is finite} \qquad \text{(X30-3)}$$

$$T(x,0)=T_{in} \qquad \text{(X30-4)}$$

Dimensionless groups for the **X30B1T0** problem

$$\tilde{T}_{\text{X30B1T0}}=\frac{T(x,t)-T_{in}}{T_f-T_{in}},\ \tilde{q}=\frac{q(x,t)}{h_1(T_f-T_{in})},\ \tilde{x}=\frac{x}{L},\ \ \tilde{t}=\frac{\alpha t}{L^2},\ B_1=\frac{h_1L}{k} \qquad \text{(X30-5)}$$

Dimensionless mathematical description of the X30B1T0 problem

$$\frac{\partial^2\tilde{T}}{\partial\tilde{x}^2}=\frac{\partial\tilde{T}}{\partial\tilde{t}},\quad 0<\tilde{x}<\infty,\ \tilde{t}>0 \qquad \text{(X30-6)}$$

$$-\frac{\partial\tilde{T}}{\partial\tilde{x}}(0,\tilde{t})=B(1-\tilde{T}(0,t)) \qquad \text{(X30-7)}$$

$$\tilde{T}(\infty,\tilde{t}) \text{ is finite} \qquad \text{(X30-8)}$$

$$\tilde{T}(\tilde{x},0)=0 \qquad \text{(X30-9)}$$

The temperature and heat flux solutions are

$$\tilde{T}_{\text{X30B1T0}}(\tilde{x},\tilde{t})=\text{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right)\text{-}e^{B_1\tilde{x}+B_1^2\tilde{t}}\text{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}+B_1\sqrt{\tilde{t}}\right) \qquad \text{(X30-10a)}$$

(X30-10b)

Dimensionless groups for the **X30B0T1** problem

$$\tilde{T}_{X30B0T1}=\frac{T(x,t)-T_f}{T_{in}-T_f},\ \tilde{q}=\frac{q(x,t)}{h_1(T_{in}-T_\infty)},\ \tilde{x}=\frac{x}{L},\ \ \tilde{t}=\frac{\alpha t}{L^2},\ B_1=\frac{h_1L}{k} \qquad \text{(X30-11)}$$

Dimensionless mathematical description of the **X30B1T0** problem

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} = \frac{\partial \tilde{T}}{\partial \tilde{t}}, \quad 0 < \tilde{x} < \infty, \ \tilde{t} > 0 \tag{X30-12}$$

$$-\frac{\partial \tilde{T}}{\partial \tilde{x}}(0,\tilde{t}) = -B\tilde{T}(0,t) \tag{X30-13}$$

$$\tilde{T}(\infty,\tilde{t}) \text{ is finite} \tag{X30-14}$$

$$\tilde{T}(\tilde{x},0) = 1 \tag{X30-15}$$

The temperature and heat flux solutions are

$$\tilde{T}_{\text{X30B0T1}}(\tilde{x},\tilde{t}) = \text{erf}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right) + e^{B_1\tilde{x}+B_1^2\tilde{t}}\text{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}} + B_1\sqrt{\tilde{t}}\right) \tag{X30-16a}$$

Note that

$$\tilde{T}_{\text{X30B1T0}}(\tilde{x},\tilde{t}) + \tilde{T}_{\text{X30B0T1}}(\tilde{x},\tilde{t}) = 1 \tag{X30-17}$$

X40B1T00, see C&J page 306

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \quad 0 < x < \infty, \ t > 0 \tag{X40-1}$$

$$-k\frac{\partial T}{\partial x}(0,t) + (\rho c_p)_1 L \frac{\partial T}{\partial t}(0,t) = q_0 \tag{X40-2}$$

$$T(\infty,t) \text{ is finite} \tag{X40-3}$$

$$T(x,0) = T_{in} \tag{X40-4}$$

Dimensionless groups to reduce to the X40B1T0 problem

$$\tilde{T}_{X40B1T0}(\tilde{x},\tilde{t}) = \frac{T(x,t) - T_{in}}{q_0 L / k}, \ \tilde{q} = \frac{q(x,t)}{q_0}, \ \tilde{x} = \frac{x}{L}, \ \tilde{t} = \frac{\alpha t}{L^2}, \ P = \frac{(\rho c_p)_1}{\rho c_p} \tag{X40-5}$$

Dimensionless mathematical description of the X40B1T0 problem

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} = \frac{\partial \tilde{T}}{\partial \tilde{t}}, \quad 0 < \tilde{x} < \infty, \ \tilde{t} > 0 \tag{X40-6}$$

$$-\frac{\partial \tilde{T}}{\partial \tilde{x}}(0,\tilde{t}) + P\frac{\partial \tilde{T}}{\partial \tilde{t}} = 1 \tag{X40-7}$$

$$\tilde{T}(\infty,\tilde{t}) \text{ is finite} \tag{X40-8}$$

$$\tilde{T}(\tilde{x},0) = 0 \tag{X40-9}$$

Solution for the X40B1T0 problem

$$\tilde{T}_{X40B1T0}(\tilde{x},\tilde{t}) = \sqrt{4\tilde{t}}\,\text{ierfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right) - P\ \text{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right) + Pe^{\frac{\tilde{x}}{P}+\frac{\tilde{t}}{P^2}}\text{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}} + \frac{\sqrt{\tilde{t}}}{P}\right) \tag{X40-10}$$

X50B1T00, see C&J page 307