The other departure functions can be derived from these. Note the mathematical similarity between $P$ in the pressure dependent formulas and $ρ$ in the density-dependent formulas.

Example 7.5 Application of pressure-dependent formulas in compression of methane

Methane gas undergoes a continuous throttling process from upstream conditions of 40°C and 20 bars to a downstream pressure of 1 bar. What is the gas temperature on the downstream side of the throttling device? An expression for the molar heat capacity of methane in the ideal gas state is

$$C_p = 19.25 + 0.0523 \cdot T + 1.197E-5 \cdot T^2 - 1.132E-8 \cdot T^3; \ T \ [\text{K}]; \ C_p \ [\text{J/mol-K}]$$

The virial equation of state may be used for methane at these conditions:

$$Z = 1 + BP/RT = 1 + (B^0 + \omega B^1)P_r/T_r$$

where $B^0 = 0.083 - 0.422/T_r^{1.6}$ and $B^1 = 0.139 - 0.172/T_r^{4.2}$

**Solution:** Since a throttling process is isenthalpic, the enthalpy departure will be used to calculate the outlet temperature.

$$\Delta H = 0 = H_2 - H_1 = (H_2 - H_2^{ig}) + (H_2^{ig} - H_1^{ig}) - (H_1 - H_1^{ig})$$

The enthalpy departure for the first and third terms in paranthesis on the right had side can be calculated using Eqn. 7.31. For the integrand, the temperature derivative of $Z$ is required. Recognizing $B$ is a function of temperature only, performing the integrations,

$$\left(\frac{\partial Z}{\partial T}\right)_p = 1 + \frac{P}{R}\left[\frac{\partial(B \cdot (1/T))}{\partial T}\right]_p = \frac{P}{R}\left[\frac{1}{T} \frac{dB}{dT} - \frac{B}{T^2}\right]$$

Inserting the derivative into Eqn 7.31

$$\left(\frac{H - H^{ig}}{RT}\right)_0 = \int_0^P \left[\frac{B}{T} - \frac{dB}{dT}\right] dP$$

$$\left(\frac{H - H^{ig}}{RT}\right)_0 = \frac{P}{R} \left(\frac{B}{T} - \frac{dB}{dT}\right)$$

$$\left(\frac{S - S^{ig}}{R}\right) = -\frac{P}{R} \frac{dB}{dT}$$

We can easily show

$$\frac{dB^0}{dT_r} = \frac{0.6752}{T_r^{2.6}} \quad \frac{dB^1}{dT_r} = \frac{0.7224}{T_r^{5.2}}$$
Section 7.7 Reference States

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If we wish to calculate state changes in a property, then the reference state is not important, and all reference state information drops out of the calculation. However, if we wish to generate a chart or table of thermodynamic properties, or compare our calculations to a thermodynamic table/chart, then designation of a reference state becomes essential. Also, if we need to solve unsteady-state problems, the reference state is important because the answer may depend on the reference state as shown in Example 2.13 on page 72. The quantity $H_R - U_R = (PV)_R$ is non-zero, and although we may substitute $(PV)_R = RT_R$ for an ideal gas, for a real fluid we must use $(PV)_R = Z_R RT_R$, where $Z_R$ has been determined at the reference state. We also may use a real fluid reference state or an ideal gas reference state. Whenever we compare our calculations with a thermodynamic chart/table, we must take into consideration any differences between our reference state and that of the chart/table. Therefore, to specify a reference state for a real fluid, we need to specify:

Pressure