3.11 TURBINE CALCULATIONS

The following example shows calculations using steam tables for the various turbine outlet states that can occur.

Example S3.1 Turbine Outlet Calculations

An adiabatic turbine inlet (state 1) is 500°C and 1.4 MPa. For each of the following outlet conditions (state 2), determine the specified quantities.

(a) P_2 = 0.6 MPa, η = 0.85. Find H_2, S_2, T_2.

(b) P_2 = 0.03 MPa, η = 0.85. Find H_2, S_2, T_2.

(c) P_2 = 0.01 MPa, η = 0.9. Find H_2, S_2, T_2.

(d) P_2 = 0.01 MPa, q = 0.99. Find η.

Soution: First, the inlet properites are determined, H_1 = 3474.8 kJ/kg, S_1 = 7.6047 kJ/kg-K. The reversible calculation is perfomed for each outlet condition, recognizing that a reversible turbine is isentropic.

(a) $S_2' = S_1$ = 7.6047 kJ/kg-K. Comparing with S^{satV}= 6.7593 kJ/kg-K at P_2 = 0.6 MPa, $S_2' > S^{satV}$, so the reversible outlet state is superheated. Interpolating:

T (°C}	*H* (kJ/kg)	*S* (kJ/kg-K)
350	3166.1	7.5481
		7.6047
400	3270.8	7.7097

$$H_2' = 3166.1 + \frac{7.6047 - 7.5481}{7.7097 - 7.5481}(3270.8 - 3166.1) = 3202.8 \text{ kJ/kg}$$

by similar interpolation, T_2' = 367.5°C.

$$\Delta H' = 3202.8 - 3474.8 = -272.0 \text{ kJ/kg}$$

Applying η calculation, $\Delta H = \eta\,\Delta H'$ = 0.85(–272) = –231.2 kJ/kg,

$$H_2 = H_1 + \Delta H = 3474.8 - 231.2 = 3243.6 \text{ kJ/kg}$$

T (°C}	*H* (kJ/kg)	*S* (kJ/kg-K)
350	3166.1	7.5481
	3243.6	
400	3270.8	7.7097

$$S_2 = 7.5481 + \frac{3243.6 - 3166.1}{3270.8 - 3166.1}(7.7097 - 7.5481) = 7.6677 \text{ kJ/kg-K}$$

by similar interpolation, T_2 = 387.0°C

Note: The reversible and actual outlets are both one-phase in part (a). Also, $S_2 > S_2' = S_1$ and $H_2 > H_2'$ which are always true for irreversible turbines. $T_2 > T_2'$, which is a general result for one-phase output.

Example S3.1 Turbine Outlet Calculations

(b) $S_2' = S_1$ = 7.6047 kJ/kg-K. Comparing with S^{satV}= 7.7675 kJ/kg-K at P_2 = 0.03 MPa, $S_2' < S^{satV}$, so the reversible outlet state is two-phase. Interpolating at T_2' = 69.1°C

$$q' = \frac{7.6047 - 0.9441}{6.8234} = 0.976$$

using Eqn 1.22

$$H_2' = 289.27 + 0.976(2335.28) = 2568.5 \text{ kJ/kg}$$

$$\Delta H' = 2568.5 - 3474.8 = -906.3 \text{ kJ/kg}$$

Applying η calculation, $\Delta H = \eta\, \Delta H'$ = 0.85(–906.3) = –770.35 kJ/kg,

$$H_2 = H_1 + \Delta H = 3474.8 - 770.35 = 2704.4 \text{ kJ/kg}$$

Comparing with H_2 with H^{satV}= 2624.55 kJ/kg at P_2 = 0.03 MPa, $H_2 > H^{satV}$, so the outlet state is superheated. A double interpolation is required. Performing the first interpolation between 0.01 and 0.05 MPa will bracket the outlet state. (Note: 0.03 MPa is halfway between 0.01 and 0.05 MPa, so tabulated values are obtained by averaging rather than by a slower interpolation.)

T (°C}	H (kJ/kg)	S (kJ/kg-K)
100	(2687.5 + 2682.4)/2 = 2684.95	(8.4489 + 7.6953)/2 = 8.0721
	2704.4	
150	(2783.0 + 2780.2)/2 = 2781.6	(8.6892 + 7.9413)/2 = 8.3153

Interpolating:

$$S_2 = 8.0721 + \frac{2704.4 - 2684.95}{2781.6 - 2684.95}(8.3153 - 8.0721) = 8.121 \text{ kJ/kg-K}$$

Simlarly, by interpolation, T_2 = 110.1°C

Note: The reversible state is two-phase, and the actual outlet is one-phase for part (b). Also, $S_2 > S_2' = S_1$ and $H_2 > H_2'$ which are always true for irreversible turbines. $T_2 > T_2'$, which is a general result for one-phase output.

(c) $S_2' = S_1$ = 7.6047 kJ/kg-K. Comparing with S^{satV}= 8.1488 kJ/kg-K at P_2 = 0.01 MPa, $S_2' < S^{satV}$, so the reversible outlet state is two-phase. Interpolating at T_2' = 45.81°C

$$q' = \frac{7.6047 - 0.6492}{7.4996} = 0.927$$

using Eqn 1.22

$$H_2' = 191.81 + 0.927(2392.05) = 2409.2 \text{ kJ/kg}$$

$$\Delta H' = 2409.2 - 3474.8 = -1065.6 \text{ kJ/kg}$$

Applying η calculation, $\Delta H = \eta\, \Delta H'$ = 0.90(–1065.6) = –959.0 kJ/kg,

$$H_2 = H_1 + \Delta H = 3474.8 - 959.0 = 2515.8 \text{ kJ/kg}$$

Example S3.1 Turbine Outlet Calculations

Comparing H_2 with H^{satV}= 2583.86 kJ/kg at P_2 = 0.01 MPa, $H_2 < H^{satV}$, so the outlet state is two-phase. Using saturation properties along with H_2:

$$q = \frac{2515.8 - 191.81}{2392.05} = 0.972$$

using Eqn 1.22

$$S_2 = 0.6492 + 0.972(7.4996) = 7.9388 \text{ kJ/kg}$$

The actual outlet is wet steam at T_2 = 45.81°C. The reversible outlet and the actual outlet are both wet steam for part (c). Also, $S_2 > S_2' = S_1$ and $H_2 > H_2'$ which are always true for irreversible turbines. For two-phase output, $T_2 = T_2'$, however $q_2 > q_2'$ which is a general result for a two-phase outlet.

(d) The first part of the calculation is the same as part (c)

$$H_2' = 2409.2 \text{ kJ/kg}, \Delta H' = -1065.6 \text{ kJ/kg}$$

For the actual outlet, using Eqn 1.22,

$$H_2' = 191.81 + 0.99(2392.05) = 2560.4 \text{ kJ/kg}$$

$$\Delta H = 2560.4 - 3474.8 = -914.4 \text{ kJ/kg}$$

$$\eta = -914.4/(-1065.6) = 0.86$$