

**Errata List For**  
**NONLINEAR SYSTEMS – First Edition**  
**Updated on March 3, 2004**  
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The errors reported here are for the first printing of the first edition, which was published in June, 1991. Many of the errors were corrected in the second printing.

In the dedication, change “Koktović” to “Kokotović”.

### **Chapter 1**

1. Page 3, Line 1: Change “the the” to “the”.
2. Page 3, Line 13: Change “the roots” to “the real roots”.
3. Page 15, Line 10: Change to “If  $h(\cdot)$  is differentiable ...”
4. Page 18, Line 18: Change “Section 5.3” to “Section 5.2”.
5. Page 24, Line 10: Change “Analysis” to “analysis”.
6. Page 38, Line 3 from the bottom: Change “a node” to “a node with distinct eigenvalues”. When  $A$  has multiple nonzero real eigenvalues, infinitesimally small perturbations could result in a pair of complex eigenvalues. Hence, a stable (respectively, unstable) node would either remain a stable (respectively, unstable) node or become a stable (respectively, unstable) focus.
7. Page 39, Line 7 of the 2nd paragraph: Change “eigenvalues” to “eigenvalue”.
8. Page 46, Line 5 of the 2nd paragraph: Change “a node” to “a node with distinct eigenvalues”.
9. Page 47, First line after Example 1.5: Change “forgoing” to “foregoing”.
10. Page 51, Figure caption: Change “negative” to “positive”.
11. Page 54, Figure 1.37 (b): reverse the arrow heads.
12. Page 58, Exercise 1.11:  $\omega$  is the angular velocity.
13. Page 60, Exercise 1.14: Change “For each for” to “For each of”.
14. Page 60, Exercise 1.15: Change “For each for” to “For each of”.

### **Chapter 2**

1. Page 65, Line 6 of the 2nd paragraph: Change “eigenvalue” to “eigenvalue of”.
2. Page 67, Section 2.1.2: Let  $f(x)$  be continuously differentiable throughout the section.
3. Page 68, Line 4: Change “ $f : R^n \rightarrow R^m$  is differentiable” to “ $f : R^n \rightarrow R$  is continuously differentiable”. The mean value theorem is valid only for a scalar function of a vector argument. It was erroneously stated for a vector function of a vector argument. At every point in the book where the mean value theorem is used, it should be applied component wise, leading to the same conclusions. Corrections will be made at the respective pages.
4. Page 70, Replace Line 19 by:  $0 \cdot x = 0$ ,  $\alpha(x + y) = \alpha x + \alpha y$ , and  $(\alpha + \beta)x = \alpha x + \beta x$ , for all  $x, y \in \mathcal{X}$ .
5. Page 71, Line 2 from the bottom: Change “ $x_k(t) - x$ ” to “ $x_k(t) - x(t)$ ”.
6. Page 72, Line 4 of the Proof: Change “ $\leq$ ” to “ $=$ ”.

7. Page 77, Line 5: Correct the spelling of the third occurrence of Lipschitz.
8. Page 77, Line 2 from the bottom: Change “ $\|\partial f/\partial x\|$ ” to “ $\|\partial f/\partial x\|_\infty$ ”.
9. Page 78, Change the first three lines to:  
 $z_i$  on the line segment joining  $x$  and  $y$  such that

$$|f_i(t, x) - f_i(t, y)| = \left| \frac{\partial f_i}{\partial x}(t, z_i)(x - y) \right| \leq L_0 \|x - y\|_\infty$$

Hence

$$\|f(t, x) - f(t, y)\|_\infty \leq L_0 \|x - y\|_\infty$$

See Lemma 3.1 of the third edition for a different proof that allows  $L_0$  to be a bound on any  $p$ -norm of the Jacobian matrix, not just the infinity norm.

10. Page 85, Line 4 from the bottom: Change “ $\leq$ ” to “ $=$ ”.
11. Page 91, Exercise 2.2: on the left-hand side of the second inequality, replace  $\frac{1}{n}$  by  $\frac{1}{\sqrt{n}}$ . On the left-hand side of the third inequality, replace  $\frac{1}{m}$  by  $\frac{1}{\sqrt{m}}$ .
12. Page 94, Exercise 2.2, Line 2: Change “ $f(x)$ ” to “ $f(t, x)$ ”.
13. Page 94, Exercise 2.2, Part (5): Change “Section 1.1.5” to “Section 1.1.6”.

### Chapter 3

1. Page 100, Line 18: Change “ $R^n$ ” to “ $R$ ”.
2. Page 100, Line 19: Change “differentiable” to “continuously differentiable”.
3. Page 101, last line of the text and first line of the footnote: Change “entirely inside” to “in the interior of”.
4. Page 104, Line 3: replace the phrase “which is ... (nonnegative).” with a new sentence “Also,  $V(x)$  is positive definite if and only if all the leading principal minors of  $P$  are positive”. The corresponding statement for positive semidefinite matrices is only necessary.
5. Page 110, Line 3 of the second paragraph: Change “converges” to “converge”.
6. Page 113, line after (3.8): Change “open set” to “set”.
7. Page 115, Line 9: Change  $\frac{1}{2}k$  to  $1/(2k)$ .
8. Page 116, First line of Lemma 3.1: Change “bounded” to “bounded and belongs to  $D$ ”.
9. Page 117, Line 2: Change “ $\Omega$ ” to “ $\Omega \subset D$ ”.
10. Page 117, Line 10 from the bottom: Remove the sentence “However, checking ... principal axes”. This sentence is wrong. See Exercise 4.9 of the third edition.
11. Page 118, last line: Change “principle” to “principle”.
12. Page 120, Line 9 from the bottom: Change “ $R^n$ ” to “ $R^2$ ”.
13. Page 121, last line: Insert the integral sign,  $\int_0^{x_i}$ .
14. Page 124, Line 7 of the Proof: Change “order o f” to “order of”.
15. Page 129: Change the last equation to

$$f_i(x) = f_i(0) + \frac{\partial f_i}{\partial x}(z_i) x$$

where  $z_i$  is a point ...

16. Page 130: Change the first eight lines to  
 $x$  to the origin lies entirely in  $D$ . Since  $f(0) = 0$ , we can write  $f_i(x)$  as

$$f_i(x) = \frac{\partial f_i}{\partial x}(z_i)x = \frac{\partial f_i}{\partial x}(0)x + \left[ \frac{\partial f_i}{\partial x}(z_i) - \frac{\partial f_i}{\partial x}(0) \right] x$$

Hence

$$f(x) = Ax + g(x)$$

where

$$A = \frac{\partial f}{\partial x}(0), \quad \text{and} \quad g_i(x) = \left[ \frac{\partial f_i}{\partial x}(z_i) - \frac{\partial f_i}{\partial x}(0) \right] x$$

The function  $g_i(x)$  satisfies

$$|g_i(x)| \leq \left\| \frac{\partial f_i}{\partial x}(z_i) - \frac{\partial f_i}{\partial x}(0) \right\| \|x\|$$

17. Page 130: Change the last sentence before Theorem 3.7 to “The theorem is known as *Lyapunov’s indirect method*.”
18. Page 132, Line 18: Change “in unstable” to “is unstable”.
19. Page 133, Line 3: Change “ $\|z\|$ ” to “ $\|z\|_2$ ”.
20. Page 138, Line 11: Change “ $x \in R^n$ ” to “ $x \in D$ ”.
21. Page 138, Example 3.16, second equation: Change “ $\dot{x}_1$ ” to “ $\dot{x}_2$ ”.
22. Page 149, Line 3 of the footnote 18: Change “it continuous” to “it is continuous”.
23. Page 153, Exercise 3.6, part (a): Change “ $x \in R^2$ ” to “ $x \in R^2, x \neq 0$ ”.
24. Page 154, Exercise 3.10: Change “a function  $V_1(x)$ ” to “a continuously differentiable function  $V_1(x)$ ”.
25. Page 158, Line 4: Change “differentiable” to “continuously differentiable”.
26. Page 159, Exercise 3.23: The set  $G$  is a simply connected domain containing a neighborhood of the origin.

#### Chapter 4

1. Page 164, The third line after (4.1): Change “at 0” to “at  $t = 0$ ”.
2. Page 170, Line 3 of footnote: Change “Suppose” to “suppose”.
3. Page 173, Lines 1 and 2: Remove “piecewise”.
4. Page 177, Line 2 from the bottom: Change “a Lyapunov” to “Lyapunov”.
5. Page 178: In Line 10 replace  $L$  by  $\frac{L}{\sqrt{n}}$ . In Lines 10, 12, 13, 15, and 16, replace  $f$  and  $z$  by  $f_i$  and  $z_i$ , respectively. In Line 14, replace  $f(t, x)$  by  $f_i(t, x)$ . Replace Lines 17 to 22 by  
Hence

$$f(t, x) = A(t)x + g(t, x)$$

where

$$A(t) = \frac{\partial f}{\partial x}(t, 0) \quad \text{and} \quad g_i(t, x) = \left[ \frac{\partial f_i}{\partial x}(t, z_i) - \frac{\partial f_i}{\partial x}(t, 0) \right] x$$

The function  $g(t, x)$  satisfies

$$\begin{aligned} \|g(t, x)\|_2 &\leq \left( \sum_{i=1}^n \left\| \frac{\partial f_i}{\partial x}(t, z_i) - \frac{\partial f_i}{\partial x}(t, 0) \right\|_2^2 \right)^{1/2} \|x\|_2 \\ &\leq L \|x\|_2 \end{aligned}$$

6. Page 195, Line 16: Change “3.06” to “3.026”.
7. Page 195, Line 9 from the bottom: Change “a closed” to “closed”.
8. Page 199, Line 8: Change “ $\leq$ ” to “=”.
9. Page 200, Line 15: Change “Exercises” to “Exercise”
10. Page 202, Definition 4.4:  $b$ ,  $c$ , and  $T$  are positive constants.
11. Page 202, The second line after (4.21): Change “exists” to “exist”.
12. Page 203, Line 13: remove “ $t_0(\dots)$  or”.
13. Page 205, Line 22: Change “ $\|x_2\|^2$ ” to “ $\|x\|_2^2$ ”.
14. Page 206, Line 10: Change “pertubed” to “perturbed”.
15. Page 207, Line 10: Change “anlysis” to “analysis”.
16. Page 207, Line 21: Change “asymptotical” to “asymptotically”.
17. Page 207, Line 27: Change “pertubed” to “perturbed”.
18. Page 210, Line 4 from the bottom: Change  $\Delta F$  and  $f$  to  $\Delta F_i$  and  $f_i$ .
19. Page 218, Line 7: Change “ $1 < k < n$ ” to “ $1 \leq k < n$ ”.
20. Page 221: Remove Footnote 18. See Corollary 8.1 of the third edition.
21. Page 222: Change the footnote to “The function  $\nu(y, w)$  is continuously differentiable every where around the origin, except on the manifold  $w = 0$ . Both  $\nu(y, w)$  and  $\dot{\nu}(y, w)$  are defined and continuous around the origin. It can be easily seen that the statement of Theorem 3.1 is still valid”.
22. Page 225, Exercise 4.3: Change “ $R^n$ ” to “ $R$ ”.
23. Page 227, Last line: Change “ $\alpha(\|x(t_0)\| \exp$ ” to “ $\alpha(\|x(t_0)\|) \exp$ ”.
24. Page 235, Line 8: Change the third term of  $V(x)$  from  $\frac{1}{3}x_3^2$  to  $\frac{1}{2}x_3^2$  and the fourth term from  $x_1x_2$  to  $x_1x_3$ .

## Chapter 5

1. Page 238, Line 6: Change “Lipchiz” to “Lipschitz”.
2. Page 241, Line 19 Change “ $\psi(\sigma)K$ ” to “ $\psi^T(\sigma)K$ ”.
3. Page 245, Example 5.1: Change “system the” to “the system”.
4. Page 250, Line 9: Change “disk  $(-0.25, 0.25)$ ” to “disk  $D(-0.25, 0.25)$ ”.
5. Page 263, Line 12: Change “ $A^{(p)}$ ” to “ $A^{(2^p)}$ ”.
6. Page 266, line 8: Change “immdeiately” to “immediately”.
7. Page 266, line 21: Change “ $\|z^{(k+1)} - z^{(j)}\|$ ” to “ $\|z^{(k+1)} - z^{(j)}\|_2$ ”.
8. Page 266, Line 22: Change “ $g(z)$ ” to “ $\|g(z)\|_2$ ”.
9. Page 267, Line 2: Change “ $s_k \rightarrow -\epsilon$ ” to “ $s_k \rightarrow s_\infty \geq -\epsilon$ ”.
10. Page 269, Last line: Change the lower limit of the summation from  $i = 1$  to  $j = 1$ .

11. Page 272: Change the third line to

$$g_1(x_2) = \ell^T x_2, \quad \ell^T = [1 \quad 1 \quad 1 \quad 1], \quad g_2(x_1) = \eta^T x_1$$

12. Page 272, Line 11: Change “ $\gamma_{12} = 1$ ” to “ $\gamma_{12} = \|\ell\|_2 = 2$ ”.

13. Page 272, Line 14: Change the (1,2) element of the matrix  $S$  from “ $-1$ ” to “ $-2$ ”, and change the following inequality to

$$\|\eta\|_2 < \frac{1}{2}\zeta\rho_{min}$$

14. Page 282, Line 16: Change “form a” to “is a”.

15. Page 286, Line 3: Change “ $\frac{3}{2}\|z\|_2^2$ ” to “ $\frac{\sqrt{10}}{2}\|z\|_2$ ”.

16. Page 286, Line 4: Change “ $\|z\|_2^2$ ” to “ $\frac{\sqrt{5}}{2}\|z\|_2$ ”.

17. Page 292, Line 18: Change “ $-a \leq x_2 \leq a$ ” to “ $-a < x_2 < a$ ”.

18. Page 294, Line 12: Change the first term on the right-hand side from “ $MAM^{-1}x$ ” to “ $MAM^{-1}z$ ”.

19. Page 295, First equation: Change the last element of the right-hand side column from  $\alpha(T(y))/\beta(T(y))$  to  $-\alpha(T(y))/\beta(T(y))$ .

20. Page 295, the stack of equations in the middle: Change the last equation from

$$\frac{\partial T_n}{\partial y} f(y) = \alpha/\beta \quad \text{to} \quad \frac{\partial T_n}{\partial y} f(y) = -\alpha/\beta$$

21. Page 296, Equation (5.61): Change the numerator of the expression for  $\alpha$  from  $(\partial T_n/\partial y)f(y)$  to  $-(\partial T_n/\partial y)f(y)$ .

22. Page 298, Line 8, the equation for  $\alpha$ : Multiply the right-hand side on the first line by a minus sign and remove the minus sign from the denominator of the second line.

23. Page 299, Last line: Change “ $\dot{y}_3$ ” to “ $\ddot{y}_2$ ”.

24. Page 309, Last line: Change “ $=$ ” to “ $\leq$ ”.

25. Page 311, Line 15: Change “small it” to “small as it”.

26. Page 315: The Kalman-Yakubovich-Popov lemma is proved in the book only for minimal realizations. In the adaptive control problem, the realization  $(A_m, b_m, c_m^T)$  could be nonminimal. An extension due to Miller (see [83, Section 2.6]) shows that the last two equations on page 315 are valid for nonminimal realizations.

27. Page 319, Equation (5.83): Insert “ $d\tau$ ” in the integral.

28. Page 326, Lines 1–2: Change Theorem 4.8 to Theorem 5.8.

29. Page 332, The hint of Exercise 5.15: Change “the principal” to “the leading principal”.

30. Page 332, Exercise 5.16: Change the inequality satisfied by  $V_i$  to

$$c_{i1}\|x_i\|^2 \leq V_i(t, x_i) \leq c_{i2}\|x_i\|^2$$

31. Page 332, Exercise 5.19: Change the lower limit of the summation from “ $j = 1$ ” to “ $j = 1; j \neq i$ ”.

32. Page 338, Line 3, Part (b): Change  $\rho_1 = \sqrt{2}$  to  $\rho_1 = 2\sqrt{2}$ .

33. Page 338, Exercise 5.38: Change  $\psi(x)$  to  $\psi(t, x)$ .

## Chapter 6

1. Page 356, Figure 6.7: Change  $\Psi(\sin \theta)$  to  $\Psi(a \sin \theta)$ .
2. Page 363, Line 2: Change “resonable” to “reasonable”.

## Chapter 7

1. Page 388, Line 1: Change “structre” to “structure”.
2. Page 388, Line 2 of the 2nd paragraph: Change “Tayolr” to “Taylor”.
3. Page 395, Line 12: Change “ $e_0(t)$ ” to “ $e_0(t, \epsilon)$ ”.
4. Page 404, Line 5: Change “ $r_0 < r$ ” to “ $r_0 \leq r$ ”.
5. Page 412, Line 24: Change “when  $g(t, 0, \epsilon)$ ” to “when  $f(0) = 0$  and  $g(t, 0, \epsilon)$ ”.
6. Page 417, Line 6: Change “Theorems 7.1–7.3 can be ...” to “The analysis leading to Theorems 7.1 and 7.2 can be ...”.
7. Page 417, First line of Theorem 7.4: Change “has” to “have”.
8. Page 424, Line 13: Insert  $d\phi$  in the integral.
9. Page 429, Line 4 from the bottom: Change “ $O(\alpha(\epsilon))$ ” to “ $O(\alpha(\epsilon))$  close”.

## Chapter 8

1. Page 437, Line 17: Change “singualr” to “singular”.
2. Page 446, Line 14: remove the sentence “The origin  $y = 0$  is also an equilibrium point for the nonautonomous system (8.12)”.
3. Page 447, Equation (8.16): Change “ $-c$ ” to “ $-c < 0$ ”.
4. Page 448, Equation (8.21) and Page 473, Equation (8.49): Change “ $\hat{y}(t/\epsilon)$ ” to “ $\hat{y}((t - t_0)/\epsilon)$ ”.
5. Page 449, Lines 7, 9, 12, and 14: Change “ $\eta_0 - \zeta_0$ ” to “ $\eta_0 + \zeta_0$ ”.
6. Page 450, Line 12:  $\gamma = \frac{\tan^{-1} \rho}{\rho}$ .
7. Page 451, Line 1: Change “ $x > 0$ ” to “ $x \geq k > 0$ ”.
8. Page 452, Line 4 from the bottom: Change “the the” to “the”.
9. Page 462, Equation (8.38): Change  $\zeta_1(y)$  and  $\zeta_2(y)$  to  $\zeta_1(\|y\|)$  and  $\zeta_2(\|y\|)$ , respectively.
10. Page 463, Line 6 from the bottom: Change “ $\psi_1^2(y)$ ” to “ $\psi_2^2(y)$ ”.
11. Page 463, Line 5 from the bottom: Change “ $\leq$ ” to “ $=$ ”.
12. Page 471, Line 8 from the bottom: Change )) to ).
13. Page 472, Lines 9 and 12: Change “ $x(t, \epsilon) - \bar{x}(t)$ ” to “ $\|x(t, \epsilon) - \bar{x}(t)\|$ ”.
14. Page 474, Line 17: Change “a given” to “given”.
15. Page 474, Line 6 from the bottom: Change  $y_p$  to  $y_m$ .
16. Page 475, Line 20: Change “ $k_p(z - h(t, x))$ ” to “ $K(z - h(t, x))$ ” where  $K = [k_p, 0, 0]^T$ .
17. Page 481, Exercise 8.19: Change  $\psi(x)$  and  $\phi(y)$  to  $\psi_1(x)$  and  $\psi_2(y)$ , respectively.
18. Page 483, Line 3: Change “current” to “the current”.

## Appendix A to the End

1. Page 488, Figure A.2: Change “ $c_0$ ” to “ $c_0^*$ ”.
2. Page 489, Lines 1 and 2: Change “ $R_{n-1}$ ” to “ $R^{n-1}$ ”.
3. Page 490, Line 10: Change “ $(sI - A_m)$ ” to “ $(sI - A_m)^{-1}$ ”.
4. Page 491, Line 11: Change “sequences  $\{\tau_i\}$ ” to “sequence  $\{\tau_i\}$ ”.
5. Page 495, Lines 7: Change “ $t_0 \leq t \leq t_0 + \bar{T}(\eta)$ ” to “ $t_0 \leq t < t_0 + \bar{T}(\eta)$ ”.
6. Page 498: replace lines 2 to 5 by

$$H(s) = \frac{\exp[-\eta^{-1}(s)]}{h(\eta^{-1}(s))}, \quad s \geq 0$$

Since  $\eta^{-1}$  is continuous and  $h$  is positive,  $H(s)$  is continuous on  $0 < s < \infty$ , while  $\eta^{-1}(s) \rightarrow \infty$  as  $s \rightarrow 0^+$ . Hence,  $H(s)$  defines a class  $\mathcal{K}$  function on  $[0, \infty)$ . It follows that the integral

7. Page 502: In the proof of the center manifold theorem, the definition of the set  $S$  should be modified such that the functions  $\eta(y)$  have the additional property that their partial derivatives are Lipschitz in  $y$ . See Appendix C.15 of the third edition. See also the errata sheet of that edition.
8. Page 507: Change Line 6 to

$$\pi_i(t, y, \eta) = \pi_i(t, y, 0) + \pi_{iy}(t, \zeta)y = y_i + \pi_{iy}(t, \zeta)y$$

9. Page 508, Line 5 from the bottom: Change “ $g_1$ ” to “ $g_{1i}$ ”.
10. Page 508, Line 2 from the bottom: Change “ $(k_1 + k_2k_3)$ ” to “ $k_4$ ”.
11. Page 515, Line 1: Change “ $t + \delta$ ” to “ $t + \mu$ ”.
12. Page 524: Change Equation (B.29) to

$$P_1g\psi y_1 - P_1g\psi(y_1 + y_h) = 2Re[G(j\omega)a_1\delta\Psi \exp(j\omega t)]$$

13. Page 525, Line 3 from the bottom: Change “ $\|\psi_T(y_1 + y^{(2)}) - \psi_T(y_1 + y^{(1)})\|$ ” to “ $\|\psi_T(y_1 + y^{(2)}) - \psi_T(y_1 + y^{(1)})\|^2$ ”.
14. Page 527, Line 4 from the bottom: Change “ $d[\eta(\cdot, \mu), p, D]$ ” to “ $d[\eta(\cdot, \mu), D, p]$ ”.
15. Page 533, Lines 1 and 5: Change “ $\leq$ ” to “ $<$ ”.
16. Page 533: Change the second term on the right-hand side of the last line to

$$\left[ \theta_1\epsilon(t_1 - t_0) + \frac{\theta_2\epsilon\mu}{\alpha} \right]$$

17. Page 535, Line 5: Change “ $G$ ” to “ $G_i$ ”.
18. Page 536, Line 6 from the bottom: Change the left-hand side to  $\|v(\tau)\|$ .
19. Page 542, Line 12 from the bottom: Change “Lemma 4.6” to “Lemma 4.5”.
20. Page 542, Line 7 from the bottom: Change “Section 4.1” to “Section 4.5.1”.
21. Page 542, Line 3 from the bottom and Page 544, Line 1: Change “Coreless” to “Corless”.
22. Page 558, Line 4 from the bottom: Change “end” to “end of”.