Additional Exercises for Chapter 2

1. For each of the following systems, find all equilibrium points and determine the type of each isolated equilibrium.

$$(\mathbf{1}) \qquad \dot{x}_1 = x_2, \qquad \dot{x}_2 = -x_1 + x_1^3/6 - x_2$$

$$(\mathbf{2}) \qquad \dot{x}_1 = -x_1 + x_2, \qquad \dot{x}_2 = 0.1x_1 - 2x_2 - x_1^2 - 0.1x_1^3$$

$$(\mathbf{3}) \qquad \dot{x}_1 = x_2, \qquad \dot{x}_2 = -x_1 + x_2(1 - 3x_1^2 - 2x_2^2)$$

$$(\mathbf{4}) \qquad \dot{x}_1 = -x_1 + x_2(1 + x_1), \qquad \dot{x}_2 = -x_1(1 + x_1)$$

2. Find all equilibrium points of the system

$$\dot{x}_1 = ax_1 - x_1x_2, \qquad \dot{x}_2 = bx_1^2 - cx_2$$

for all positive real values of a, b, and c, and determine the type of each equilibrium.

3. For each of the following systems, construct the phase portrait and discuss the qualitative behavior of the system.

$$(\mathbf{1}) \qquad \dot{x}_1 = x_2, \qquad \dot{x}_2 = x_1 - 2\tan^{-1}(x_1 + x_2)$$

$$(\mathbf{2}) \qquad \dot{x}_1 = x_2, \qquad \dot{x}_2 = -x_1 + x_2\left(1 - 3x_1^2 - 2x_2^2\right)$$

$$(\mathbf{3}) \qquad \dot{x}_1 = 2x_1 - x_1x_2, \qquad \dot{x}_2 = 2x_1^2 - x_2$$

4. Consider the system

$$\dot{x}_1 = -(x_1 - x_1^2) + 1 - x_1 - x_2, \qquad \dot{x}_2 = -(x_2 - x_2^2) + 1 - x_1 - x_2$$

(a) Find all equilibrium points and determine the type of each point.

(b) Construct the phase portrait and discuss the qualitative behavior of the system.