- Claim (Proof of Theorem 4.1, page 116): By repetition of of previous arguments, we know that for every $a > 0$, we can choose $b > 0$ such that $\Omega_b \subset B_a$.

  Proof: It is enough to consider $a < r$. Let $\gamma = \min_{a \le \|x\| \le r} V(x)$ and take $b < \gamma$. Then, $\Omega_b = \{x \in B_r \mid V(x) \le b\}$ is in the interior of B_a. If this was not the case, there would be a point $p \in \Omega_b$ that lies in the region $a \le \|x\| \le r$. At this point $V(p) \ge \gamma > b$, which contradicts that fact that for $x \in \Omega_a$, $V(x) \le a < \gamma$.

- Claim (page 662): $\psi(s)$ is continuous, positive definite, and increasing. There is a class $\mathcal{K}$ function $\alpha_1(s)$ such that $\alpha_1(s) \le k\psi(s)$ with $0 < k < 1$.

  Proof: Take
$$\alpha_1(s) = \frac{ks}{s+1}\psi(s), \qquad \text{for } s \ge 0$$
  α_1 is strictly increasing because $s/(s+1)$ is strictly increasing and ψ is increasing and positive.
$$\frac{s}{s+1} \le 1 \ \Rightarrow\ \alpha_1(s) \le k\psi(s)$$

- Claim (page 664): $\bar{\delta}(\varepsilon)$ is positive definite, nondecreasing, but not necessarily continuous. There is a class $\mathcal{K}$ function $\zeta(r)$ such that $\zeta(r) \le k\bar{\delta}(r)$ with $0 < k < 1$.

  Proof: Note from the last line of page 663 that $\bar{\delta}(\varepsilon) \le \varepsilon$. Let
$$\zeta(s) = k\int_0^s e^{-\sigma}\bar{\delta}(\sigma)\, d\sigma$$
  Because $\bar{\delta}$ is monotone, it is Riemann integrable. Because the product of two Riemann integrable functions is Riemann integrable, $e^{-\sigma}\bar{\delta}(\sigma)$ is Riemann integrable.
$$\zeta(s) - \zeta(r) = k\int_r^s e^{-\sigma}\bar{\delta}(\sigma)\, d\sigma$$
  ζ is continuous because
$$|\zeta(s) - \zeta(r)| \le k\varepsilon|r - s|$$
  ζ is strictly increasing because $e^{-\sigma}\bar{\delta}(\sigma) > 0$ for all $\sigma > 0$. Finally, because $\bar{\delta}$ is nondecreasing
$$\bar{\delta}(\sigma) \le \bar{\delta}(s^-) \le \bar{\delta}(s) \le \bar{\delta}(s^+), \quad \text{for } 0 < \sigma < s$$
  where $\bar{\delta}(s^-)$ and $\bar{\delta}(s^+)$ are the left and right limits of $\bar{\delta}$ at s.
$$\zeta(s) \le k\int_0^s e^{-\sigma}\bar{\delta}(s)\, d\sigma = k(1 - e^{-s})\bar{\delta}(s) \le k\bar{\delta}(s)$$